Witness of mixed separable states useful for entanglement creation

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Absolutely separable states form a special subset of the set of all separable states, as they remain separable under any global unitary transformation unlike other separable states. We consider the set of absolutely separable bipartite states and show that it is convex and compact in any arbitrary dimensional Hilbert space. Through a generic approach of construction of suitable Hermitian operators we prove the completeness of the separation axiom for identifying any separable state that is not absolutely separable. We demonstrate the action of such witness operators in different qudit systems. Examples of mixed separable systems are provided, pointing out the utility of the witness in entanglement creation using quantum gates. Decomposition of witnesses in terms of spin operators or photon polarizations facilitates their measurability for qubit states.

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I. INTRODUCTION

The problem whether a quantum state is separable or entangled remains one of the most involved problems in quantum information science, which is underlined by the observation that the separability problem is NP-hard [1]. A pure quantum state $|\psi\rangle$ is separable if it can be written in the product form as $|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle$. A mixed quantum state ϱ_{sep} is separable if it can be written as $\varrho_{\text{sep}} = \sum_{i=1}^{k} p_i |e_i, f_i\rangle\langle e_i, f_i|$, where $|e_i, f_i\rangle$ are product states and $p_i \geqslant 0, \sum_{i=1}^{k} p_i = 1$. States which are not separable are called entangled. In lower dimensions, specifically in $2 \otimes 2$ and $2 \otimes 3$, a state is separable if and only if it has a positive partial transpose [2,3]. However, in higher dimensions there exist entangled states with positive partial transpose [4].

The theory of entanglement witnesses [3,5,6] provides a useful procedure to check whether a state is entangled. Entanglement witnesses W are Hermitian operators with at least one negative eigenvalue and satisfy the inequalities (i) $\text{Tr}(W\varrho_{\text{sep}}) \geqslant 0, \forall$ separable states ϱ_{sep} and (ii) $\text{Tr}(W\varrho_{\text{ent}}) < 0$ for at least one entangled state ϱ_{ent}. Entanglement witnesses can be used to detect the presence of entanglement experimentally [6–8]. The strength of the theory of entanglement witnesses is also due to its completeness, which asserts that if a state is entangled there will always be a witness that detects it [3].

Entanglement witnesses (EW) constitute an application of a more general framework of the geometric form of the celebrated Hahn-Banach theorem in functional analysis [9]. The theorem states that if a set is convex and compact, then any point lying outside the set can be separated by a hyperplane. The separation axiom has been also utilized in the inception of teleportation witnesses [10], which identify useful entangled states for quantum teleportation. Further work in the direction of constructing optimal [11] and complete [12] teleportation witnesses has also been performed, analogous to entanglement witnesses.

An intriguing feature of the set of separable states is concerning the problem of separability from spectrum [13]. This problem calls for a characterization of those separable states σ for which $U\sigma U^\dagger$ is also separable for all unitary operators U. A possible approach towards this end is to find constraints on the eigenvalues of σ such that it remains separable under any factorization of the corresponding Hilbert space. The states that are separable from spectrum are also termed as absolutely separable states [14]. There exists a ball of known radius centered at the maximally mixed state $\frac{1}{mn}(I \otimes I)$ (for $mn \times mn$ density matrices), where all the states within the ball are absolutely separable [15]. However, there exist absolutely separable states outside this ball too [16].

The problem of separability from spectrum was first handled in the case of $2 \otimes 2$ systems [17], where it was shown that σ is absolutely separable if and only if (iff) its eigenvalues (in descending order) satisfy $\lambda_1 \leqslant \lambda_3 + 2\sqrt{\lambda_2\lambda_4}$. A closely related problem is the characterization of the states which have positive partial transpose (PPT) from spectrum, i.e., the states σ_{ppt} with the property that $U\sigma_{ppt}U^\dagger$ is PPT for any unitary operator U. It was shown [18] that $\sigma_{ppt} \in D(H_2 \otimes H_n)$ [$D(X)$ represents the bounded linear operators acting on X] is PPT from spectrum iff its eigenvalues obey $\lambda_1 \leqslant \lambda_{2n-1} + 2\sqrt{\lambda_{2n-2}\lambda_{2n}}$. It has been recently shown that separability from spectrum is equivalent to PPT from spectrum for states living in $D(H_2 \otimes H_n)$ [19].

The generation of entanglement from separable states is one of the leading experimental frontiers at present [20]. As absolutely separable states remain separable under global unitary operations, such states cannot be used as input states for entanglement creation. Though pure product states are not absolutely separable, the same is not true for mixed separable states which become absolutely separable after crossing a given amount of mixedness [21]. Given the ubiquity of environmental interactions in turning pure states into mixed ones, it is of practical importance to determine whether a state is eligible to be used as input for entanglement generation. The utility of mixed separable states which are not absolutely separable was highlighted in [16] for the generation of maximally entangled mixed states. Mixed separable states from which entanglement can be created have also been studied in other works [22].

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Quantum gates have been employed to generate entanglement, especially in the context of quantum computation where unitary gates operate on qubits to perform information processing. Much work has been devoted to studying the entangling capacity of unitary gates [23,24]. Quantum algorithms use pure product states which can be turned into maximally entangled states using global unitary operations. However, if the state is maximally mixed no benefit can be drawn from it through global unitary operations. States in some vicinity of the maximally mixed state also remain separable as noted in [15], though such states may have possible implications in nuclear magnetic resonance quantum computation [25]. However, states not close to the maximally mixed state may be useful for entanglement creation. So, it is important to study what happens when one moves from one extreme of a maximally mixed state to the other, i.e., a pure product state, within the set of all separable states.

Given the immense significance of mixed separable states, we present here systematic proposal to identify separable states which are not absolutely separable. Our approach is somewhat different from the objective of imposing restrictions on the spectrum of absolutely separable states [17–19]. Our motivation here is to identify those separable states which are not absolutely separable, i.e., the separable states χ for which $U\chi U^\dagger$ is entangled for some unitary operator U. To this end, we characterize the set of all absolutely separable states in any finite-dimensional bipartite system as convex and compact. This enables one to construct Hermitian operators which identify separable states that are not absolutely separable in any arbitrary dimension Hilbert space. Proposing a general method of construction of witnesses, we illustrate their action on various two-qudit systems. Examples of unitary operations presented here include the celebrated CNOT (controlled-NOT) gate. We further show that the witnesses can be decomposed in terms of spin operators and locally measurable photon polarizations for qubit states, in order to facilitate their experimental realization.

II. EXISTENCE, CONSTRUCTION, AND COMPLETENESS OF WITNESS

We begin with some notations and definitions needed. We consider density matrices in any arbitrary dimensional bipartite system, i.e., $\rho \in D(H_m \otimes H_n)$. $\mathbf{S} = \{\rho : \rho \text{ is separable}\}$ is the set of all separable states, and $\mathbf{AS} = \{\sigma \in \mathbf{S} : U\sigma U^\dagger \text{ is separable } \forall \text{ unitary operators } U\}$ is the set of all absolutely separable states. **AS** forms a nonempty subset of **S**, as $\frac{1}{mn}(I \otimes I) \in \mathbf{AS}$. A point x is called a limit point of a set A if each open ball centered on x contains at least one point of A different from x. The set is closed if it contains each of its limit points [26].

Theorem. **AS** is a convex and compact subset of **S**.

Proof. **AS is convex**: let $\sigma_1,\sigma_2 \in \mathbf{AS}$ and $\sigma = \lambda\sigma_1 + (1-\lambda)\sigma_2$, where $\lambda \in [0,1]$. Consider an arbitrary unitary operator U. Therefore,

$$U\sigma U^\dagger = \lambda U\sigma_1 U^\dagger + (1-\lambda)U\sigma_2 U^\dagger = \lambda\sigma_1' + (1-\lambda)\sigma_2', \quad (1)$$

where $\sigma_i' = U\sigma_i U^\dagger, i = 1,2$. $\sigma_1',\sigma_2' \in \mathbf{S}$ as $\sigma_1,\sigma_2 \in \mathbf{AS}$. Since **S** is convex, $U\sigma U^\dagger \in \mathbf{S}$, which implies that $\sigma \in \mathbf{AS}$. Hence **AS** is convex.

AS is compact: consider an arbitrary limit point θ of **AS** (**AS** will always have a limit point; for example, in the neighborhood of the identity there are other absolutely separable states). The same θ must also be a limit point of **S** as $\mathbf{AS} \subset \mathbf{S}$. Thus $\theta \in \mathbf{S}$, because **S** is closed. Now, let us inductively construct a sequence $\{\theta_n\}$ of distinct states from **AS** such that $\theta_n \to \theta$ as follows:

$$\begin{aligned}
&\theta_1 \in B_1(\theta) \cap \mathbf{AS}, \quad \theta_1 \neq \theta,\\
&\theta_2 \in B_{\frac{1}{2}}(\theta) \cap \mathbf{AS}, \quad \theta_2 \neq \theta,\theta_1\\
&\ldots \in \ldots\ldots\ldots\ldots\\
&\ldots \in \ldots\ldots\ldots\ldots\\
&\theta_n \in B_{\frac{1}{n}}(\theta) \cap \mathbf{AS}, \quad \theta_n \neq \theta,\theta_1,\theta_2\ldots\theta_{n-1}.
\end{aligned} \quad (2)$$

Here $B_r(\theta)$ denotes an open ball of radius r centered at θ. (This construction is possible because each neighborhood of θ contains infinitely many points of **AS**, θ being a limit point of **AS**.) For the above mentioned choice of θ_n's, evidently $\theta_n \to \theta$. Hence, for any unitary operator U, one has $U\theta_n U^\dagger \to U\theta U^\dagger$. Thus $U\theta_n U^\dagger \in \mathbf{S}$ for each $n \geqslant 1$, as $\theta_n \in \mathbf{AS}$. Since **S** is a closed set, it must contain the limit of the sequence $\{U\theta_n U^\dagger\}$, which is $U\theta U^\dagger$. Hence $U\theta U^\dagger \in \mathbf{S}$, for any arbitrary choice of the unitary operator U. Therefore, $\theta \in \mathbf{AS}$ as we already have $\theta \in \mathbf{S}$. Since θ is an arbitrary limit point of **AS**, it follows that **AS** contains all its limit points, thereby implying that **AS** is closed [26]. As any closed subset of a compact set is compact [26], one concludes that **AS** is compact because **S** is compact. Hence the theorem. ■

In view of the theorem above, we now formally define a Hermitian operator T which identifies separable but not absolutely separable states through the following two inequalities:

$$\mathrm{Tr}(T\sigma) \geqslant 0, \quad \forall \sigma \in \mathbf{AS}, \quad (3)$$

$$\exists \chi \in \mathbf{S} - \mathbf{AS}, \quad \text{such that } \mathrm{Tr}(T\chi) < 0. \quad (4)$$

Therefore, T identifies those separable states χ that become entangled under some global unitary operation.

Consider $\chi \in \mathbf{S} - \mathbf{AS}$. There exists a unitary operator U_e such that $U_e\chi U_e^\dagger$ is entangled. Consider an entanglement witness W that detects $U_e\chi U_e^\dagger$, i.e., $\mathrm{Tr}(WU_e\chi U_e^\dagger) < 0$. Using the cyclic property of the trace, one obtains $\mathrm{Tr}(U_e^\dagger W U_e \chi) < 0$. It follows that

$$T = U_e^\dagger W U_e \quad (5)$$

is our desired operator. Next, considering its action on an arbitrary absolutely separable state σ, we see that $\mathrm{Tr}(T\sigma) = \mathrm{Tr}(U_e^\dagger W U_e \sigma) = \mathrm{Tr}(WU_e\sigma U_e^\dagger)$. As σ is absolutely separable, $U_e\sigma U_e^\dagger$ is a separable quantum state, and since W is an entanglement witness, $\mathrm{Tr}(WU_e\sigma U_e^\dagger) \geqslant 0$. This implies that T has a non-negative expectation value on all absolutely separable states σ. The completeness of the separation axiom follows from the completeness of entanglement witness, viz., for any entangled state $U_e\chi U_e^\dagger$, there always exists a witness W [3]. Thus, if χ is a separable but not absolutely separable state, then one can always construct an operator T in the above-mentioned procedure which distinguishes χ from absolutely separable states.

III. ILLUSTRATIONS

As the first example, consider the separable state in $D(H_2 \otimes H_2)$ given by [21]

$$\chi_{2\otimes 2} = \frac{1}{4}\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}, \tag{6}$$

which becomes entangled on application of the unitary operator

$$U_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & \sqrt{2} & 0 & 0 \\ 0 & 0 & \sqrt{2} & 0 \\ -1 & 0 & 0 & 1 \end{pmatrix}. \tag{7}$$

The entanglement witness

$$W_1 = \begin{pmatrix} c^2 & 0 & 0 & 0 \\ 0 & 0 & -c & 0 \\ 0 & -c & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{8}$$

with $c = \frac{1}{\sqrt{2}+1}$ detects the entangled state $U_1 \chi_{2\otimes 2} U_1^\dagger$. Hence the operator

$$T_1 = U_1^\dagger W_1 U_1 \tag{9}$$

gives $\mathrm{Tr}(T_1 \chi_{2\otimes 2}) < 0$, detecting $\chi_{2\otimes 2}$ to be a state which is not absolutely separable.

Next, consider the following separable density matrix $\chi_{2\otimes 4} \in D(H_2 \otimes H_4)$:

$$\chi_{2\otimes 4} = \begin{pmatrix} 1/4 & 0 & 1/4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/4 & 0 & 1/4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/4 & 0 & 1/4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/4 & 0 & 1/4 \end{pmatrix}. \tag{10}$$

The state $\chi^e_{2\otimes 4} = U_2 \chi_{2\otimes 4} U_2^\dagger$ is entangled due to the unitary operator

$$U_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & \sqrt{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sqrt{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{2} & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \tag{11}$$

Therefore, the operator $T_2 = U_2^\dagger W_2 U_2$ detects the state $\chi_{2\otimes 4}$ as a separable but not absolutely separable state, where W_2 is the entanglement witness for the entangled state $\chi^e_{2\otimes 4}$, given by $W_2 = Q^{T_B}$, with Q being a projector on $|10\rangle - |01\rangle$.

It is hard to classify states separable from spectrum in dimensions other than $2 \otimes n$, due to the absence of suitable methodology in the existing literature. However, through our approach of witnesses we can identify states which are not absolutely separable in any arbitrary dimension. Consider the isotropic state $\in D(H_3 \otimes H_3)$, given by

$$\chi_{3\otimes 3} = \alpha |\phi_3^+\rangle\langle\phi_3^+| + \frac{1-\alpha}{9} I, \tag{12}$$

where $|\phi_3^+\rangle = \frac{1}{\sqrt{3}}(|00\rangle + |11\rangle + |22\rangle)$. This state is separable for $-\frac{1}{8} \leqslant \alpha \leqslant \frac{1}{4}$ [27]. It is observed that the unitary operator $U_3 = I - (\frac{\sqrt{2}-1}{\sqrt{2}})(|00\rangle\langle 00| + |22\rangle\langle 22|) + \frac{1}{\sqrt{2}}(|00\rangle\langle 22| - |22\rangle\langle 00|)$ converts $\chi_{3\otimes 3}$ to an entangled state $\chi^e_{3\otimes 3}$ in the range $\alpha \in (\frac{1}{1+3\sqrt{2}}, \frac{1}{4}]$. So, the operator $T_3 = U_3^\dagger W_3 U_3$ detects $\chi_{3\otimes 3}$ as a state that is not absolutely separable. Here W_3 is the entanglement witness for $\chi^e_{3\otimes 3}$, given by $W_3 = (|\eta\rangle\langle\eta|)^{T_B}$ with $|\eta\rangle$ being the eigenvector of $(\chi^e_{3\otimes 3})^{T_B}$ corresponding to the eigenvalue $-\frac{1}{9}\alpha + \frac{1}{9} - \frac{\sqrt{2}}{3}\alpha$.

Let us now present a construction of the witness operator for general qudit states. Take the unitary operator

$$U_{d\otimes d} = I - \left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)A + \frac{1}{\sqrt{2}}B, \tag{13}$$

where $A = |00\rangle\langle 00| + |d-1, d-1\rangle\langle d-1, d-1|$ and $B = |00\rangle\langle d-1, d-1| - |d-1, d-1\rangle\langle 00|$, and the mixed separable state

$$\chi_{d\otimes d} = \tfrac{1}{4}|00\rangle\langle 00| + \tfrac{3}{4}|d-1, d-1\rangle\langle d-1, d-1|. \tag{14}$$

The state $U_{d\otimes d}\chi_{d\otimes d}U^\dagger_{d\otimes d}$ is entangled as detected by the witness $W_{d\otimes d} = \frac{1}{d}I - |P\rangle\langle P|$, where P is the projector on the maximally entangled state $\frac{1}{\sqrt{d}}\sum_{i=0}^{d-1}|ii\rangle$. Therefore, in $d \otimes d$ dimensions the operator $T_{d\otimes d} = U^\dagger_{d\otimes d} W_{d\otimes d} U_{d\otimes d}$ detects $\chi_{d\otimes d}$ as a state which is not absolutely separable.

IV. ENTANGLEMENT CREATION USING QUANTUM GATES

Let us now consider some examples of unitary quantum gates which can produce entanglement by acting on bipartite separable states. Since the construction presented above is valid for any arbitrary dimension, let us consider a case in $d_1 \otimes d_2$ dimensions where $d_1 \neq d_2$. Consider the two qudit hybrid quantum gate U_H acting on $d_1 \otimes d_2$ dimensions, whose action is defined by

$$U_H |m\rangle \otimes |n\rangle = |m\rangle \otimes |m - n\rangle, \tag{15}$$

with $m \in \mathbb{Z}_{d_1}, n \in \mathbb{Z}_{d_2}$ [23]. Let us take the initial mixed separable state

$$\chi_{d_1\otimes d_2} = \tfrac{1}{4}\chi_x + \tfrac{3}{4}\chi_y, \tag{16}$$

where χ_x is a projector on $\frac{1}{\sqrt{2}}(|0, d_2 - 1\rangle + |1, d_2 - 1\rangle)$ and χ_y a projector on $|d_1 - 1, d_2 - 1\rangle$. The state $U_H \chi_{d_1\otimes d_2} U_H^\dagger$ is entangled as identified by the witness $W_{d_1\otimes d_2} = X^{T_B}$ (X being the projector on $|02\rangle - |11\rangle$). Hence $T_{d_1\otimes d_2} = U_H^\dagger W_{d_1\otimes d_2} U_H$ detects $\chi_{d_1\otimes d_2}$ as a state which is not absolutely separable. The above example again illustrates the fact that one can construct a Hermitian operator for two qudits (for equal or different dimensions) that can recognize useful separable states from which entanglement can be created between the two qudits using global unitary operations.

We finally consider the example of the much discussed CNOT gate. The CNOT gate can generate entanglement between two qubits, if the state under consideration is not absolutely separable. If we now consider the action of U_{CNOT} on a class of mixed separable states of two qubits of the form

$$\chi_{\text{mix}} = a|00\rangle\langle 00| + b|00\rangle\langle 10| + b|10\rangle\langle 00| + (1-a)|10\rangle\langle 10|, \tag{17}$$

where $a,b \in \mathbb{R}$, we find that the states of the form $\chi^e_{\text{mix}} = U_{\text{CNOT}}\chi_{\text{mix}}U^\dagger_{\text{CNOT}}$ can be entangled. Such entanglement can be detected by the witness $W_{\text{CNOT}} = [(|10\rangle - |01\rangle)(\langle 10| - \langle 01|)]^{T_B}$. A Hermitian operator T_{CNOT} constructed according to our prescription, which detects χ_{mix} as a state not absolutely separable, is given by $T_{\text{CNOT}} = U^\dagger_{\text{CNOT}} W_{\text{CNOT}} U_{\text{CNOT}}$. Now, $\text{Tr}(T_{\text{CNOT}}\chi_{\text{mix}}) = -2b$, implying that for $b > 0$ the operator detects the class of states as useful for entanglement creation under the action of the CNOT gate. For example, for $a = 3/4$ and $b = 1/4$, we get a state that is not absolutely separable detected by the witness T_{CNOT}. On the other hand, a state of the form [19]

$$\sigma = \frac{1}{11}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 3 & 2 & 0 \\ 0 & 2 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{pmatrix} \tag{18}$$

leads to $\text{Tr}(T_{\text{CNOT}}\sigma) > 0$, remaining separable under the action of the CNOT gate, as the state σ (18) is absolutely separable. Note. though, that neither the entanglement witness W_{CNOT}, nor consequently T_{CNOT} as constructed here, are universal. As a result, the operator T_{CNOT} fails to detect some states which are not absolutely separable that exist even for $b < 0$ in the class of states (17). However, through our generic approach, one can construct another suitable witness operator to identify states not absolutely separable in the latter range.

V. DECOMPOSITION OF THE WITNESS OPERATOR

For the purpose of experimental determination of the expectation value of a witness operator on a given state, it is helpful to decompose it in terms of spin matrices [28]. As an example, the witness T_{CNOT} which detects the class of states χ_{mix} (17) as not absolutely separable admits the decomposition $T_{\text{CNOT}} = \frac{1}{2}(I\otimes I - I\otimes Z - X\otimes Z - X\otimes I)$, where X, Z are the usual Pauli spin matrices. Further, in order that the witness operator can be measured locally, it may be decomposed in the form $T = \sum_{i=1}^k c_i|e_i\rangle\langle e_i| \otimes |f_i\rangle\langle f_i|$ [28]. Experimental realization of entanglement witnesses has been achieved using polarized photon states [7]. In the case of the operator T_{CNOT}, the decomposition in terms of photon polarization states is given by

$$T_{\text{CNOT}} = |HV\rangle\langle HV| + |VV\rangle\langle VV| - |DH\rangle\langle DH| + |FH\rangle\langle FH|, \tag{19}$$

where $|H\rangle = |0\rangle$, $|V\rangle = |1\rangle$, $|D\rangle = \frac{|H\rangle+|V\rangle}{\sqrt{2}}$, and $|F\rangle = \frac{|H\rangle-|V\rangle}{\sqrt{2}}$ are the horizontal, vertical, and diagonal polarization states, respectively [7]. The above decomposition suggests a realizable method to experimentally verify whether it is possible to create an entangled state from a mixed separable state through the action of an entangling gate.

VI. SUMMARY

In this work we have proposed a framework to distinguish separable states that remain separable from those that become entangled due to global unitary operations in any arbitrary dimensional Hilbert space [13–19]. To this end we have characterized the set of all absolutely separable bipartite states as convex and compact, enabling one to construct suitable Hermitian operators for identification of states that are not globally separable. A generic procedure for construction of such operators in any dimensions is suggested, which underlines the completeness of the separation, viz., if χ is not absolutely separable then there will always be an operator which detects it. The action of the operator is demonstrated on states in various dimensions. Observational feasibility of witnesses for qubit states is highlighted through decomposition in terms of locally measurable photon polarizations.

The generation of entanglement from separable initial states is of prime importance in information processing applications [20]. Our method helps to identify eligible input states for entanglement creation using global unitary operations in general, and may be of specific relevance in quantum gate operations [24] widely used in quantum computation. Though pure product states can be readily entangled through such operations, the inevitability of environmental influences makes the consideration of mixed states highly relevant, and thereby lends practical significance to our proposal for detection of separable mixed states useful for production of entanglement. Formulations for constructing common and optimal witnesses analogously to the case of entanglement witnesses [29], as well as extensions of our scheme for multipartite states, would be of much relevance.

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