

# Evolution of primordial black holes in Jordan–Brans–Dicke cosmology

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## ABSTRACT

We consider the evolution of primordial black holes in a generalized Jordan–Brans–Dicke cosmological model where both the Brans–Dicke scalar field and its coupling to gravity are dynamical functions determined from the evolution equations. The evaporation rate for the black holes is different from the case in standard cosmology. We show that the accretion of radiation can proceed effectively in the radiation-dominated era. It follows that the black hole lifetime shortens for low initial mass, but increases considerably for larger initial mass, thus providing a mechanism for the survival of primordial black holes as candidates of dark matter. We derive a cut-off value for the initial black hole mass, below which primordial black holes evaporate out in the radiation-dominated era, and above which they survive beyond the present era.

**Key words:** black hole physics – cosmology: theory.

## 1 INTRODUCTION

The Jordan–Brans–Dicke (JBD) (Brans & Dicke 1961) theory is one of the earliest and most well-motivated alternatives to the Einstein theory of gravitation. The value of the gravitational ‘constant’ is set by the inverse of a time-dependent classical scalar field with a coupling parameter  $\omega$ . General relativity is recovered in the limit of  $\omega \rightarrow \infty$ . Solar System observations impose lower bounds on  $\omega$  (Bertotti, Less & Tortora 2003; Will 2006). Generalized JBD models are obtained in the low energy limit of higher dimensional theories. String theoretic (Gasperini & Veneziano 2007) and Kaluza–Klein (Appelquist, Chodos & Freund 1987) models after compactification of the extra dimensions yield several variants of JBD models (or general scalar-tensor models) in which the scalar field coupling  $\omega$  may become dynamical, and also models with potential terms for the JBD scalar field.

JBD models have been used for tackling several cosmological problems pertaining to different eras of evolution of the Friedman–Robertson–Walker (FRW) universe. The scenario of extended inflation (Mathiazhagan & Johri 1984; La & Steinhardt 1989) was proposed within the context of JBD cosmology. Generalized JBD models resulting from the compactification of higher dimensional actions (Majumdar & Sethi 1992; Majumdar, Seshadri & Sethi 1993; Majumdar 1997) enable efficient transition to the post-inflationary radiation-dominated phase (or reheating). The JBD scalar field has also been incorporated in quintessence scenarios for obtaining the present acceleration of the universe (Bertolami & Martins 2000; Banerjee & Pavon 2001; Sen & Seshadri 2003). A more general class of JBD theory where the coupling parameter  $\omega$  is an arbitrary function of the scalar field (Bergmann 1968, 1970; Wagoner 1970) has interesting consequences on cosmic evolution (Sahoo & Singh

2002, 2003), and has also been applied in the context of obtaining singularity free cosmology (Kalyana Rama 1997) and relic gravitational waves (Barrow, Mimoso & de Garcia Maia 1993; Singh & Sahoo 2004).

Primordial black holes (PBHs) are potentially fascinating cosmological tools (Carr 2003). It is possible for PBHs to impact through their evaporation products diverse cosmological processes such as baryogenesis and nucleosynthesis (Carr 1976; Zeldovich et al. 1977; Miyama & Sato 1978; Chechetkin et al. 1982; Barrow et al. 1991; Majumdar, Das Gupta & Saxena 1995; Liddle & Green 1998), the cosmic microwave background radiation (Chapline 1975; MacGibbon & Carr 1991) and the growth of perturbations as well (Afshordi, McDonald & Spergel 2003). PBHs could act as seeds for structure formation (Mack, Ostriker & Ricotti 2006) and could also form a significant component of dark matter through efficient early accretion in braneworld scenarios (Guedens, Clancy & Liddle 2002; Majumdar 2003; Majumdar & Mukherjee 2005; Majumdar, Mehta & Luck 2005). Mechanisms for growth of supermassive black holes by PBHs accreting dark energy have been proposed (Bean & Magueijo 2002), though recent results by Harada & Carr (2005a) and Harada, Maeda & Carr (2006) indicate the lack of self-similar growth of black holes by accreting quintessence.

The feasibility of black hole solutions in JBD theory was first discussed by Hawking (1972). The coexistence of black holes with a long range scalar field in cosmology has interesting consequences such as the possibility of energy exchange between the black holes and the scalar field (Zloshchastiev 2005). The black holes themselves could be used to constrain the variation of fundamental constants (MacGibbon 2007). Another interesting issue of gravitational memory of black holes in JBD theory has also been studied (Barrow 1992; Harada, Goymer & Carr 2002). The evolution of PBHs in JBD cosmology has, however, remained unexplored in the literature. Since JBD models entail modification of the standard cosmological evolution, it is expected that the evolution of PBHs in JBD

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models and their associated consequences on various cosmological processes could be substantially modified compared to the case of standard cosmology.

The purpose of this work is to perform a preliminary study of the evolution of PBHs in JBD cosmology. We consider a generalized scalar-tensor theory (Bergmann 1968, 1970; Wagoner 1970) where the coupling parameter  $\omega(\phi)$  is a function of the scalar field  $\phi$ , and is thus also time-dependent. We use a set of power-law solutions for the scalefactor, the JBD field and the coupling  $\omega$  obtained in the radiation era (Sahoo & Singh 2002, 2003), and investigate the net energy flux for the PBHs resulting from the processes of Hawking evaporation and the accretion of radiation. We show that accretion of radiation can be effective in the radiation-dominated era since the growth of PBHs is always subdominant to the growth of the cosmological horizon in this JBD model. In the matter-dominated era, the dynamics of the PBHs is solely governed by the evaporation process whose rate is determined by another set of power-law solutions of this JBD model in the matter-dominated era (Sahoo & Singh 2002, 2003). The lifetime for PBHs is obtained in terms of their initial masses. Our results outline a viable scenario for the growth and survival of PBHs as constituents of dark matter in JBD cosmology.

## 2 GENERALIZED BRANS-DICKE MODEL

We begin with a generalized JBD action given by

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[ \phi R - \frac{\omega(\phi)}{\phi} (\partial_\mu \phi)^2 \right] + S_{\text{matter}}, \quad (1)$$

where the strength of the running gravitational coupling is given by the inverse of  $\phi$ , i.e. at any time  $t$ ,  $G = 1/\phi(t)$  and  $S_{\text{matter}}$  corresponds to the action of the relativistic fluid of particles ( $p = \rho/3$ ) in the flat ( $k = 0$ ) FRW early universe with scalefactor  $a$ . The Friedman equation and the equation of motion for the JBD field  $\phi$  obtained from the above action are given respectively by

$$\frac{\dot{a}^2}{a^2} + \frac{\dot{a}\dot{\phi}}{a\dot{\phi}} - \frac{\omega\dot{\phi}^2}{6\phi^2} = \frac{\rho}{3\phi} \quad (2)$$

and

$$\ddot{\phi} + 3\frac{\dot{a}\dot{\phi}}{a} = \frac{\rho - 3p}{2\omega + 3} - \frac{\omega\dot{\phi}}{2\omega + 3} \quad (3)$$

with the energy conservation equation given by

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0. \quad (4)$$

For a radiation-dominated evolution, one has  $\rho \approx \rho_R \propto a^{-4}$  and  $p = \rho/3$ . Assuming power-law solutions for the scalefactor  $a(t)$ , the JBD field  $\phi(t)$  and also the coupling parameter  $\omega(t)$ , one can obtain (Sahoo & Singh 2002, 2003) the following time-dependences of these quantities:

$$a(t) = a_i \left( \frac{t}{t_i} \right)^{\frac{3}{\omega_i + 6}}, \quad (5)$$

$$\phi(t) = \phi_i \left( \frac{t}{t_i} \right)^{-\frac{3}{\omega_i + 6}}, \quad (6)$$

$$2\omega(t) + 3 = (2\omega_i + 3) \left( \frac{t}{t_i} \right)^{\frac{2\omega_i + 3}{\omega_i + 6}}, \quad (7)$$

where the subscript  $i$  indicates the initial values of the variables. In order to minimize the departure of the evolution of the scalefactor from the case of the radiation-dominated evolution of standard

cosmology [i.e.  $a(t) \propto t^{1/2}$ ], we choose  $|\omega_i| \ll 1$ . Without loss of generality, we set  $\phi_i = \xi M_{\text{pl}}^2$ , where  $M_{\text{pl}} = G^{-1/2}$  is the Planck mass (corresponding to the present value of  $G$ ). Note that the solution for the JBD field  $\phi$  indicates the strengthening of gravity with the increase of cosmic time. Such a behaviour for  $\phi$  arises not just from the specific model (1) used here, but is a rather generic feature of several generalized JBD or scalar-tensor models with a potential for the scalar field which rolls down the slope leading to the increase of  $G$  with time (Majumdar & Sethi 1992; Majumdar et al. 1993; Majumdar 1997).

## 3 EVOLUTION OF PRIMORDIAL BLACK HOLES

Primordial black holes could form due to various mechanisms (Carr 1985) some of which have been studied in detail in the literature (Zeldovich & Novikov 1966; Hawking 1971; Carr & Hawking 1974; Hawking, Moss & Stewart 1982; Kodama, Sasaki & Sato 1982). The gravitational collapse of perturbations within the cosmological horizon could lead to subhorizon-sized black hole formation after Jeans crossing. Such a mechanism may not be very effective, however, in JBD cosmology where the Jeans length itself could be significantly increased due to the weakening of gravity in certain models by a large value of the JBD field in the early universe. Nevertheless, a strong (first order) inflationary phase transition in extended inflation models driven by the JBD field (La & Steinhardt 1989; Majumdar & Sethi 1992; Majumdar et al. 1993; Majumdar 1997) naturally leads to unnucleated and trapped false vacuum regions and topological defects such as domain walls and wormholes that could easily collapse into black holes, and could also create superhorizon density perturbations. The formation of superhorizon scale PBHs in the expanding FRW background has been studied recently (Harada & Carr 2005b). In the present analysis, we will not take recourse to any particular formation mechanism for PBHs, but rather study their cosmological evolution in JBD theory, assuming that there exist PBHs in such scenarios.

We now consider the evolution of PBHs in the cosmological background governed by the above solutions (5), (6) and (7). We assume that the PBH density is low enough to ensure radiation domination. For a PBH immersed in the radiation field, the accretion of radiation leads to the increase of its mass with the rate given by

$$\dot{M}_{\text{acc}} = 4\pi f r_{\text{BH}}^2 \rho_R, \quad (8)$$

where  $r_{\text{BH}} = 2M/\phi$  is the black hole radius and  $f$  is the accretion efficiency. Using the solution for  $\phi$  given by equation (6) with the assumption of  $|\omega_i| \ll 1$ , and using  $\rho_R = [(3M_{\text{pl}}^2)/(32\pi t^2)]$ , one obtains

$$\dot{M}_{\text{acc}} = \frac{B M^2}{t}, \quad (9)$$

where  $B = (3f)/(2\xi^2 M_{\text{pl}}^2 t_i)$ . Equation (9) is integrated to yield

$$\frac{M}{M_i} = \frac{1}{1 - B M_i \ln(t/t_i)} \quad (10)$$

with  $M_i$  being the initial mass of the PBH at time  $t_i$ . Since the logarithmic growth rate for a PBH given by equation (10) is subdominant to the linear growth of the horizon mass  $M_{\text{H}} \sim t$  (since  $a \sim t^{1/2}$ ), once a PBH is formed, it is indeed possible for it to grow in size by accreting the radiation energy within its cosmological horizon.

For a complete picture of PBH evolution, one also needs to consider the Hawking evaporation process, whose rate is given by

$$\dot{M}_{\text{evap}} = -4\pi g r_{\text{BH}}^2 \sigma T^4, \quad (11)$$

where  $T = \phi/(8\pi M)$  is the Hawking temperature,  $\sigma$  the Stefan–Boltzmann constant and  $g$  is the effective number of degrees of freedom of the particles emitted by the black hole. The solution for  $\phi$  given by equation (6) leads to

$$\dot{M}_{\text{evap}} = -\frac{A}{M^2 t}, \quad (12)$$

where  $A = (g\sigma\xi^2 M_{\text{pl}}^4 t_i)/(256\pi^3)$ . The complete evolution for the PBH is thus described by the combination of equations (9) and (12):

$$\dot{M} = -\frac{A}{M^2 t} + \frac{BM^2}{t}. \quad (13)$$

It is apparent from equation (13) that for PBHs with initial mass  $M_i \ll (A/B)^{1/4}$ , the rate of evaporation exceeds that of accretion. For such a case, accretion soon becomes negligible and black holes lose energy at a rate given effectively by equation (12). Note that though the rate of evaporation decreases with time, it is still higher than the corresponding rate in standard cosmology. This is because the Hawking temperature  $T = \phi/(8\pi M)$  is larger for JBD PBHs for large  $\phi$ . Hence, a PBH with initial mass  $M_i < (A/B)^{1/4}$  evaporates out much quicker, with a lifetime  $t_{\text{evap}}$  given by

$$\frac{t_{\text{evap}}}{t_i} = \exp\left[\frac{1}{3A}\left(\frac{M_i}{M_{\text{pl}}}\right)^3\left(\frac{t_{\text{pl}}}{t_i}\right)\right]. \quad (14)$$

However, PBHs with initial mass  $M_i > (A/B)^{1/4}$  experience monotonic growth with accretion dominating over evaporation throughout the period of validity of equation (13), that is, throughout the radiation-dominated era. Equation (13) can be integrated exactly and leads to the following mass–time relationship for the PBHs:

$$\ln\left(\frac{t}{t_i}\right) = \frac{1}{2BC^{1/4}}\left[\tan^{-1}\left(\frac{M}{C^{1/4}}\right) - \tan^{-1}\left(\frac{M_i}{C^{1/4}}\right)\right] + \frac{1}{4BC^{1/4}}\ln\left|\frac{(M - C^{1/4})(M_i + C^{1/4})}{(M_i - C^{1/4})(M + C^{1/4})}\right| \quad (15)$$

with  $C = A/B$ . Accretion of radiation can proceed effectively till the universe stays radiation dominated, i.e. up to the era of matter radiation equality  $t_{\text{eq}}$ . This result is qualitatively different from the widely accepted picture in the standard cosmological evolution where accretion of radiation in the radiation-dominated era seems to be ineffective (Carr 2003). The domination of accretion over evaporation is observed also in other modified gravity theories, such as in the braneworld scenario (Guedens et al. 2002; Majumdar 2003; Majumdar & Mukherjee 2005; Majumdar et al. 2005). The maximum mass achieved by a PBH of initial mass  $M_i$  is given by

$$\frac{M_{\text{max}}}{M_i} \approx \frac{1}{1 - BM_i \ln(t_{\text{eq}}/t_i)}. \quad (16)$$

In the matter-dominated era,  $\rho_M \sim a^{-3}$  and  $p = 0$ . A set of solutions for the JBD cosmological equations (2)–(4) is given by Sahoo & Singh 2002 (2003)

$$a(t) = a(t_{\text{eq}})\left(\frac{t}{t_{\text{eq}}}\right)^{2/3}, \quad (17)$$

$$\phi(t) = \phi(t_{\text{eq}})\left(\frac{t}{t_{\text{eq}}}\right)^{-4/3}, \quad (18)$$

$$\omega(t) = -\omega(t_{\text{eq}})\left(\frac{t}{t_{\text{eq}}}\right)^{4/3}. \quad (19)$$

Matching these solutions with those obtained in the radiation-dominated era given by equations (5), (6) and (7), one has  $\phi(t_{\text{eq}}) = \xi M_{\text{pl}}^2(t_i/t_{\text{eq}})^{1/2}$  and  $\omega(t_{\text{eq}}) = \omega_i(t_{\text{eq}}/t_i)^{1/2}$ . Substituting the solution

for the JBD field  $\phi$  in equation (11) leads to the following evaporation law for PBHs in the matter-dominated era:

$$\dot{M} = -\tilde{A}\frac{1}{M^2 t^{8/3}} \quad (20)$$

with  $\tilde{A} = A t_i M_{\text{pl}}^4 t_{\text{eq}}^{5/3}$ . The mass of the PBHs evolve as

$$M(t) = M_{\text{max}}\left[1 - \frac{3\tilde{A}}{5M_{\text{max}}^3}(t_{\text{eq}}^{-5/3} - t^{-5/3})\right]^{1/3} \quad (21)$$

in the matter-dominated era. The lifetime for the PBHs  $t_{\text{evap}}$  is given by

$$t_{\text{evap}} = t_{\text{eq}}\left[1 - \frac{5}{3}\frac{M_{\text{max}}^3}{\tilde{A}t_{\text{eq}}^{-5/3}}\right]^{-3/5} \quad (22)$$

which upon (using equation 16) reduces to

$$t_{\text{evap}} = t_{\text{eq}}\left\{1 - \frac{5}{3}\frac{M_i^3}{[1 - BM_i \ln(t_{\text{eq}}/t_i)]^3 (At_i M_{\text{pl}}^4)}\right\}^{-3/5}. \quad (23)$$

Since the availability of background radiation diminishes substantially in the matter-dominated era, one can safely neglect any accretion term corresponding to the accretion of radiation by the PBHs. However, other forms of accretion such as of the energy of a quintessence field could, in principle, be effective (Bean & Magueijo 2002), that we do not consider in the present analysis. The rate of evaporation is again faster, as in the radiation-dominated era, than in the case of standard cosmology (where  $t_{\text{evap}} \sim M_{\text{max}}^3$ ). However, the rate decreases with time as is evident from the right-hand side of equation (20) whose counterpart in the standard theory is independent of time.

The overall lifetime of a JBD PBH has an interesting comparison to a PBH lifetime in standard cosmology. A JBD PBH with initial mass  $M_i < (A/B)^{1/4}$  evaporates out quicker than a PBH in standard cosmology, being unable to accrete in the radiation-dominated era. However, when the initial mass exceeds  $(A/B)^{1/4}$ , accretion proceeds effectively and dominates over evaporation in the radiation-dominated era increasing the PBH mass at the time of matter radiation equality. In the matter-dominated era, though the evaporation rate of JBD PBH is faster, it starts to evaporate out much later than in standard cosmology (without accretion). Thus, the lifetime for a PBH with  $M_i > (A/B)^{1/4}$  is enhanced in the JBD scenario compared to the case of standard cosmology. Note that in the present analysis we have not considered any possible back reaction of the PBHs on the local background value of the JBD field resulting from the local change of energy density  $\rho$  due to the PBHs. The form of the JBD field equation (3) is such that the  $\rho$ -dependence drops out in the radiation-dominated era. Further, in the late matter-dominated era the value of  $\omega$  is large enough (in order to satisfy present observational bounds) such that the  $\rho$ -dependent term again becomes negligible in equation (3). So any back reaction could be effective only in the early matter-dominated epoch during which a local rise in energy density could result in a slower rate of decrease for the  $\phi$  field near the PBHs than what is obtained in equation (18). The resultant change in the PBH evaporation law (20) could introduce a minor correction to the PBH lifetime (23) that we have neglected here.

It would be interesting to observe particular examples of evolving PBHs in JBD cosmology. Note that the accretion and evaporation rates depend on the initial value  $\phi$  of the JBD field through the quantities  $A$  and  $B$  defined earlier. Here, we fix the value of  $\phi_i$  or  $\xi$  by setting the present value of  $\phi(t_{\text{now}}) = M_{\text{pl}}^2$  (corresponding to the present strength of gravity) and evolving  $\phi$  given by equations (18) and (6)

corresponding to the matter- and radiation-dominated eras, respectively, backwards in time. We thus obtain the value of  $\phi(t_{\text{EW}}) = 10^{18} M_{\text{pl}}^2$ , where  $t_{\text{EW}}$  denotes the electroweak era since which the JBD evolution equations (3) are assumed to be valid. From equation (14) it follows that for a PBH with  $M_i < (A/B)^{1/4}$  to survive the radiation-dominated era,  $M_i \geq 10^{13} M_{\text{pl}}$ . A PBH which forms at time  $t_{\text{EW}}$  with an initial mass of the order of the cosmological horizon mass at  $t_{\text{EW}}$  satisfies the condition  $M_i > (A/B)^{1/4}$ , and accretes radiation. Though the actual growth of mass turns out to be negligible for such a PBH, accretion does play an important role enabling it to survive for much longer ( $t_{\text{evap}} > t_{\text{now}}$ ).

Another interesting consequence of equation (23) is that for a PBH to evaporate during the present era, i.e.  $t_{\text{evap}} = t_{\text{now}}$  one needs to have  $M_{\text{max}} \approx 10^{10} M_{\text{pl}}$ . Since accretion cannot cause significant growth, one requires  $M_i \sim M_{\text{max}} \approx 10^{10} M_{\text{pl}}$ . However, a PBH with such a small initial mass cannot even survive the radiation-dominated era. Only those PBHs which have  $M_i > 10^{13} M_{\text{pl}}$  enter the matter-dominated era and these also survive up to the present era, i.e.  $t_{\text{evap}} \gg t_{\text{now}}$ . Thus, there are no end products of black hole evaporation to contend with in JBD cosmology during the matter-dominated era and beyond up to present times. All PBHs with initial mass  $M_i < 10^{13} M_{\text{pl}}$  evaporate out in the radiation-dominated era itself. On the other hand, PBHs with larger mass can survive as intermediate mass astrophysical black holes contributing to the dark matter density.

## 4 CONCLUSIONS

To summarize, in this paper we have studied the evolution of PBHs in JBD cosmology and obtained a cut-off value for the initial mass which decides whether the PBHs could survive up to today. An interesting consequence of our analysis is that no PBHs evaporate during the matter-dominated era, and hence there is no distortion of the cosmic microwave background radiation spectrum due to evaporating PBHs. Though our results are in the context of a particular JBD model, the sort of evolution obtained for the JBD field  $\phi$  is generic to other JBD models as well, in particular, those following from higher dimensional theories (Majumdar & Sethi 1992; Majumdar et al. 1993; Majumdar 1997). In order to satisfy Solar System constraints (Will 2006) such as the one recently obtained through the frequency shift of radio photons to and from the Cassini spacecraft as they passed near the sun (Bertotti et al. 2003), one requires  $|\omega(t_{\text{now}})| > 10^4$ , which using equations (19) and (7), leads to  $\omega_i \sim 10^{-14}$ , thus strongly validating our approximation of  $|\omega_i| \ll 1$  used to derive the black hole evolution equation in the radiation-dominated era. With the possibility of future observations of actually finding or ruling out JBD models by determining  $\omega$  (Acquaviva & Verde 2007), it would indeed be exciting to rework the standard observational constraints (Carr 1985; Liddle & Green 1998; Carr 2003) on the density of PBHs at several cosmological eras in the JBD scenario. In particular, the faster evaporation of smaller PBHs in the radiation-dominated era could impose tighter constraints on the initial mass spectrum from nucleosynthesis bounds (Liddle & Green 1998). Investigation of these issues calls for a more comprehensive analysis by considering a population of black holes in the early universe evolving according to the JBD dynamics.

Post-acceptance, we came to know about earlier works on modification of PBH constraints due to gravitational memory in scalar-tensor models (Barrow 1992; Barrow & Carr 1996). While these papers have investigated the role of gravitational memory on formation and evaporation of PBHs, it might be interesting to reanalyze such studies with the inclusion of accretion in the radiation dominated era.

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