Entanglement Witness Operator for Quantum Teleportation

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The ability of entangled states to act as a resource for teleportation is linked to a property of the fully entangled fraction. We show that the set of states with their fully entangled fraction bounded by a threshold value required for performing teleportation is both convex and compact. This feature enables the existence of Hermitian witness operators, the measurement of which could distinguish unknown states useful for performing teleportation. We present an example of such a witness operator illustrating it for different classes of states.

Introduction.—Quantum information processing is now widely recognized as a powerful tool for implementing tasks that cannot be performed using classical means [1]. A large number of algorithms for various information processing tasks such as super dense coding [2], teleportation [3] and key generation [4] have been proposed and experimentally demonstrated. At the practical level information processing is implemented by manipulating states of quantum particles, and it is well known that not all quantum states can be used for such purposes. Hence, given an unknown state, one of the most relevant issues here is to determine whether it is useful for quantum information processing.

The key ingredient for performing many information processing tasks is provided by quantum entanglement. The experimental detection of entanglement is facilitated by the existence of entanglement witnesses [5,6] which are Hermitian operators with at least one negative eigenvalue. The existence of entanglement witnesses is a consequence of the Hahn-Banach theorem in functional analysis [7,8] providing a necessary and sufficient condition to detect entanglement. Motivated by the nature of different classes of entangled states, various methods have been suggested to construct entanglement witnesses [9–12]. Study of entanglement witnesses [13] has proceeded in directions such as the construction of optimal witnesses [9,11], Schmidt number witnesses [14], and common witnesses [15]. The possibility of experimental detection of entanglement through the measurement of expectation values of witness operators for unknown states is facilitated by the decomposition of witnesses in terms of Pauli spin matrices for qubits [16] and Gell-Mann matrices in higher dimensions [17]. For macroscopic systems the properties of thermodynamic quantities provide a useful avenue for detection of entanglement [18].

Teleportation [3] is a typical information processing task where at present there is intense activity in extending the experimental frontiers [19]. However, it is well-known that not all entangled states are useful for teleportation. For example, while the entangled Werner state [20] in $2 \otimes 2$ dimensions is a useful resource [21], another class of maximally entangled mixed states [22], as well as other nonmaximally entangled mixed states achieve a fidelity higher than the classical limit only when their magnitude of entanglement exceeds a certain value [23]. The problem of determining states useful for teleportation becomes conceptually more involved in higher dimensions where bound entangled states [24] also exist.

The motivation for this study is to enquire how to determine whether an unknown entangled state could be used as a resource for performing information processing tasks. In the present work we consider this question for the specific task of quantum teleportation. We propose and demonstrate the existence of measurable witness operators connected to teleportation, by making use of a property of entangled states, *viz*, the fully entangled fraction (FEF)

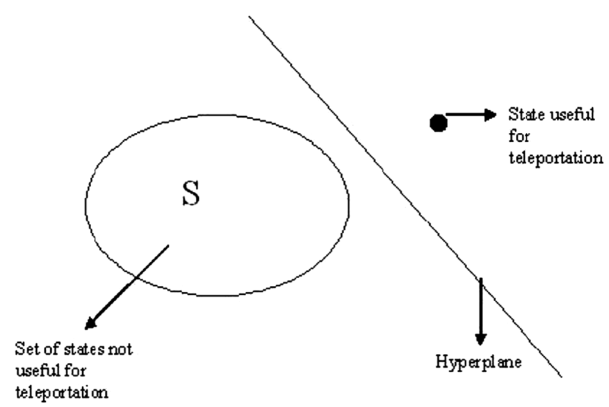

FIG. 1. The set $S = \{\rho : F(\rho) \leq \frac{1}{d}\}$ is convex and compact, and using the Hahn-Banach theorem it follows that any state useful for teleportation can be separated from the states not useful for teleportation by a hyperplane, thus providing for the existence of a witness for teleportation.

[25,26] which can be related to the efficacy of teleportation. In spite of the conceptual relevance of the FEF as a characteristic trait of entangled states [27], its actual determination could be complicated for higher dimensional systems [28,29]. Our proof of the existence of witnesses connected to a relevant threshold value for the FEF enables us to construct a suitable witness operator for teleportation, as is illustrated with certain examples.

Proof of existence of witness.—The fully entangled fraction (FEF) [26] is defined for a bipartite state ρ in $d \otimes d$ dimensions as

$$F(\rho) = \max_U \langle \psi^+ | U^\dagger \otimes I \rho U \otimes I | \psi^+ \rangle \tag{1}$$

where $|\psi^+\rangle = \frac{1}{\sqrt{d}} \sum_{i=0}^{d-1} |ii\rangle$ and U is a unitary operator. A quantum channel is useful for teleportation if it can provide a fidelity higher than what can be done classically. The fidelity depends on the FEF of the state; e.g., a state in $d \otimes d$ dimensions works as a teleportation channel if its FEF $> \frac{1}{d}$ [26–28].

Here we propose the existence of a Hermitian operator which serves to distinguish between states having FEF higher than a given threshold value from other states. FEF $> \frac{1}{d}$ is a benchmark which measures the viability of quantum states in teleportation. Let us consider the set S of states having FEF $\leq \frac{1}{d}$. A special geometric form of the Hahn-Banach theorem in functional analysis [7,8] states that if a set is convex and compact, then a point lying outside the set can be separated from it by a hyperplane. The existence of entanglement witnesses are indeed also an outcome of this theorem [5,6]. We now present the proof that the set S of states with FEF $\leq \frac{1}{d}$ is indeed convex and compact, so that the separation axiom in the form of the Hahn-Banach theorem could be applied in order to demonstrate the existence of Hermitian witness operators for teleportation.

Proposition: The set $S = \{\rho : F(\rho) \leq \frac{1}{d}\}$ is convex and compact. *Proof:* The proof is done in two steps. (i) We first show that S is convex. Let $\rho_1, \rho_2 \in S$. Therefore,

$$F(\rho_1) \leq \frac{1}{d}, \qquad F(\rho_2) \leq \frac{1}{d}. \tag{2}$$

Consider $\rho_c = \lambda \rho_1 + (1-\lambda)\rho_2$, where $\lambda \in [0,1]$ and $F(\rho_c) = \langle \psi^+ | U_c^\dagger \otimes I \rho_c U_c \otimes I | \psi^+ \rangle$. Now, $F(\rho_c) = \lambda \langle \psi^+ | U_c^\dagger \otimes I \rho_1 U_c \otimes I | \psi^+ \rangle + (1-\lambda) \langle \psi^+ | U_c^\dagger \otimes I \rho_2 U_c \otimes I | \psi^+ \rangle$. Let $F(\rho_i) = \langle \psi^+ | U_i^\dagger \otimes I \rho_i U_i \otimes I | \psi^+ \rangle$, $(i = 1, 2)$. This is possible since the group of unitary matrices is compact; hence, the maximum will be attained for a unitary matrix U. It follows that $F(\rho_c) \leq \lambda F(\rho_1) + (1-\lambda) F(\rho_2)$. Using Eq. (2) we have

$$F(\rho_c) \leq \frac{1}{d} \tag{3}$$

Thus, ρ_c lies in S, and hence, S is convex. (ii) *We now show that S is compact.* Note that in a finite dimensional Hilbert space, in order to show that a set is compact it is enough to show that the set is closed and bounded. The set S is bounded as every density matrix has a bounded spectrum, i.e., eigenvalues lying between 0 and 1. In order to prove that the set S is closed, consider first the following lemma. *Lemma*: Let A and B be two matrices of size $m \times n$ and $n \times r$ respectively. Then $\|AB\| \leq \|A\| \|B\|$, where the norm of a matrix A is defined as $\|A\| = \sqrt{\mathrm{Tr} A^\dagger A} = \sqrt{\sum_i \sum_j |A_{ij}|^2}$. *Proof of the lemma*: Let

$$A = \begin{pmatrix} A_1 \\ A_2 \\ . \\ . \\ A_m \end{pmatrix}$$

and $B = [B^{(1)} B^{(2)} \ldots B^{(r)}]$, where A_i's are row vectors of size n and $B^{(j)}$'s are column vectors of size n respectively. Using the Cauchy-Schwarz inequality, it follows that $|(AB)_{ij}| = |A_i B^{(j)}| \leq \|A_i\| \|B^{(j)}\|$. Therefore, one has

$$\|AB\|^2 = \sum_{i=1}^{m} \sum_{j=1}^{r} |(AB)_{ij}|^2 \leq \sum_{i=1}^{m} \sum_{j=1}^{r} \|A_i\|^2 \|B^{(j)}\|^2 \tag{4}$$

The r.h.s of the above inequality can be expressed as $\sum_{i=1}^m \|A_i\|^2 \sum_{j=1}^r \|B^{(j)}\|^2 = \|A\|^2 \|B\|^2$, from which it follows that $\|AB\| \leq \|A\| \|B\|$.

For any two density matrices ρ_a and ρ_b, assume the maximum value of FEF is obtained at U_a and U_b respectively, i.e., $F(\rho_a) = \langle \psi^+ | U_a^\dagger \otimes I \rho_a U_a \otimes I | \psi^+ \rangle$ and $F(\rho_b) = \langle \psi^+ | U_b^\dagger \otimes I \rho_b U_b \otimes I | \psi^+ \rangle$. Therefore, we have $F(\rho_a) - F(\rho_b) = \langle \psi^+ | U_a^\dagger \otimes I \rho_a U_a \otimes I | \psi^+ \rangle - \langle \psi^+ | U_b^\dagger \otimes I \rho_b U_b \otimes I | \psi^+ \rangle$ from which it follows that $F(\rho_a) - F(\rho_b) \leq \langle \psi^+ | U_a^\dagger \otimes I \rho_a U_a \otimes I | \psi^+ \rangle - \langle \psi^+ | U_a^\dagger \otimes I \rho_b U_a \otimes I | \psi^+ \rangle$ since $\langle \psi^+ | U_a^\dagger \otimes I \rho_b U_a \otimes I | \psi^+ \rangle \leq \langle \psi^+ | U_b^\dagger \otimes I \rho_b U_b \otimes I | \psi^+ \rangle$. Hence, $F(\rho_a) - F(\rho_b) \leq \langle \psi^+ | U_a^\dagger \otimes I (\rho_a - \rho_b) U_a \otimes I | \psi^+ \rangle$, implying

$$F(\rho_a) - F(\rho_b) \leq |\langle \psi^+ | U_a^\dagger \otimes I (\rho_a - \rho_b) U_a \otimes I | \psi^+ \rangle|. \tag{5}$$

Now, using the above lemma, one gets $F(\rho_a) - F(\rho_b) \leq \| \langle \psi^+ | \| \| U_a^\dagger \otimes I \| \| (\rho_a - \rho_b) \| \| U_a \otimes I \| \| |\psi^+\rangle \|$, or $F(\rho_a) - F(\rho_b) \leq C^2 K_1^2 \|\rho_a - \rho_b\|$, where C, K_1 are positive real numbers. The last step follows from the fact that $\| \langle \psi^+ | \| = C$. Since the set of all unitary operators is compact, it is bounded, and thus for any U, $\|U \otimes I\| \leq K_1$. Similarly $F(\rho_b) - F(\rho_a) \leq C^2 K_1^2 \|\rho_b - \rho_a\| = C^2 K_1^2 \|\rho_a - \rho_b\|$. So finally, one may write

$$|F(\rho_a) - F(\rho_b)| \leq C^2 K_1^2 \|\rho_a - \rho_b\|. \tag{6}$$

This implies that F is a continuous function. Moreover, for any density matrix ρ, with $F(\rho) \in [\frac{1}{d^2}, 1]$, one has $F(\rho) = 1$ iff ρ is a maximally entangled pure state, and $F(\rho) = \frac{1}{d^2}$ iff ρ is the maximally mixed state [28]. For the set S in our consideration $F(\rho) \in [\frac{1}{d^2}, \frac{1}{d}]$. Hence, $S = \{\rho: F(\rho) \leq \frac{1}{d}\} = F^{-1}([\frac{1}{d^2}, \frac{1}{d}])$, is closed [8]. This completes the proof of our proposition that the set $S = \{\rho: F(\rho) \leq \frac{1}{d}\}$ is convex and compact.

It now follows from the Hahn-Banach theorem [7,8], that any $\chi \notin S$ can be separated from S by a hyperplane. In other words, any state useful for teleportation can be separated from the states not useful for teleportation by a hyperplane and thus allows for the definition of a witness (Fig. 1). The witness operator, if so defined, identifies the states which are useful in the teleportation protocol, i.e., provides a fidelity higher than the classical optimum.

A witness operator for teleportation.—A Hermitian operator W may be called a teleportation witness if the following conditions are satisfied: (i) $\mathrm{Tr}(W\sigma) \geq 0$, for all states σ which are not useful for teleportation. (ii) $\mathrm{Tr}(W\chi) < 0$, for at least one state χ which is useful for teleportation. We propose a Hermitian operator for a $d \otimes d$ system of the form (using $|\psi^+\rangle = \frac{1}{\sqrt{d}} \sum_{i=0}^{d-1} |ii\rangle$)

$$W = \frac{1}{d} I - |\psi^+\rangle\langle\psi^+| \tag{7}$$

In order to prove that W is indeed a witness operator, we first show that the operator W gives a non-negative expectation over all states which are not useful for teleportation. Let σ be an arbitrary state chosen from the set S not useful for teleportation, i.e., $\sigma \in S$. Hence,

$$\mathrm{Tr}\,(W\sigma) = \frac{1}{d} - \langle\psi^+|\sigma|\psi^+\rangle \tag{8}$$

from which it follows that $\mathrm{Tr}(W\sigma) \geq \frac{1}{d} - \max_U \langle\psi^+|U^\dagger \otimes I\sigma U \otimes I|\psi^+\rangle$. Now, using the definition of the FEF, $F(\sigma)$ from Eq. (1), and the fact that $\sigma \in S$, one gets

$$\mathrm{Tr}\,(W\sigma) \geq 0 \tag{9}$$

Our task now is to show that the operator W detects at least one entangled state χ which is useful for teleportation, i.e., $\mathrm{Tr}(W\chi) < 0$, which we do by providing the following illustrations.

Let us first consider the isotropic state

$$\chi_\beta = \beta|\psi^+\rangle\langle\psi^+| + \frac{1-\beta}{d^2} I\left(-\frac{1}{d^2-1} \leq \beta \leq 1\right) \tag{10}$$

The isotropic state is entangled $\forall\ \beta > \frac{1}{d+1}$ [30]. Now, $\mathrm{Tr}(W\chi_\beta) = \frac{(d-1)(1-\beta(d+1))}{d^2}$, from which it follows that $\mathrm{Tr}(W\chi_\beta) < 0$, when $\beta > \frac{1}{d+1}$. Therefore, all entangled isotropic states are useful for teleportation. The same conclusion was obtained in Ref. [28] on explicit calculation of the FEF for isotropic states. We next consider the generalized Werner state [20,31] in $d \otimes d$ given by

$$\chi_{\mathrm{wer}} = (1-v)\frac{I}{d^2} + v|\psi_d\rangle\langle\psi_d|, \tag{11}$$

where $0 \leq v \leq 1$ and $|\psi_d\rangle = \sum_{i=0}^{d-1} \alpha_i |ii\rangle$, with $\sum_i |\alpha_i|^2 = 1$, for which one obtains $\mathrm{Tr}(W\chi_{\mathrm{wer}}) = \frac{1}{d} - \frac{1-v}{d^2} - \frac{v}{d} \sum_{i=0}^{d-1} \alpha_i \sum_{i=0}^{d-1} \alpha_i^*$. The witness W detects those Werner states which are useful for teleportation, i.e., $\mathrm{Tr}(W\rho_{\mathrm{wer}}) < 0$, which is the case when

$$\frac{1}{d} - \frac{1-v}{d^2} - \frac{v}{d} \sum_{i=0}^{d-1} \alpha_i \sum_{i=0}^{d-1} \alpha_i^* < 0. \tag{12}$$

In $2 \otimes 2$ dimensions with $\alpha_i = 1/\sqrt{2}$, one gets $\mathrm{Tr}(W\chi_{\mathrm{wer}}) = \frac{1-3v}{4} < 0$, when $v > \frac{1}{3}$. Thus, all entangled Werner states are useful for teleportation, a result which is well-known [21].

Now, consider another class of maximally entangled mixed states in $2 \otimes 2$ dimensions, which possess the maximum amount of entanglement for a given purity [22]:

$$\chi_{\mathrm{MEMS}} = \begin{pmatrix} h(C) & 0 & 0 & C/2 \\ 0 & 1-2h(C) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ C/2 & 0 & 0 & h(C) \end{pmatrix}, \tag{13}$$

where, $h(C) = C/2$ for $C \geq 2/3$, and $h(C) = 1/3$ for $C < 2/3$, with C the concurrence of χ_{MEMS}. Here we obtain $\mathrm{Tr}(W\rho_{\mathrm{MEMS}}) = \frac{1}{2} - h(C) - \frac{C}{2}$. It follows that $\mathrm{Tr}(W\rho_{\mathrm{MEMS}}) \geq 0$ when $0 \leq C \leq \frac{1}{3}$, implying that for a magnitude of the entanglement in the above range, the state χ_{MEMS} is not useful for teleportation. But, for $C > \frac{1}{3}$, the state χ_{MEMS} is suitable for teleportation, as one obtains $\mathrm{Tr}(W\rho_{\mathrm{MEMS}}) < 0$ in this case, confirming the results derived earlier in the literature [23]. However, as expected with any witness, our proposed witness operator may fail to identify certain other states that are known to be useful for teleportation. For example, the state (for $|\phi\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$ and $0 \leq a \leq 1$)

$$\rho_\phi = a|\phi\rangle\langle\phi| + (1-a)|11\rangle\langle 11| \tag{14}$$

was recently studied in the context of quantum discord [32]. This class of states is useful for teleportation but the witness W is unable to detect it as $\mathrm{Tr}(W\rho_\phi) = \frac{a}{2} \geq 0$.

Let us now briefly discuss the measurability of the witness operator. For experimental realization of the witness it is necessary to decompose the witness into operators that can be measured locally, i.e, a decomposition into projectors of the form $W = \sum_{i=1}^{k} c_i |e_i\rangle\langle e_i| \otimes |f_i\rangle\langle f_i|$ [13,16]. For implementation using polarized photons as in [33], one may take $|H\rangle = |0\rangle$, $|V\rangle = |1\rangle$, $|D\rangle = \frac{|H\rangle + |V\rangle}{\sqrt{2}}$, $|F\rangle = \frac{|H\rangle - |V\rangle}{\sqrt{2}}$, $|L\rangle = \frac{|H\rangle + i|V\rangle}{\sqrt{2}}$, $|R\rangle = \frac{|H\rangle - i|V\rangle}{\sqrt{2}}$ as the horizontal, vertical, diagonal, and the left and right circular polarization states, respectively. Using a result given in [34], our witness operator can be recast for qubits into the required form, given by

$$W = \frac{1}{2}(|HV\rangle\langle HV| + |VH\rangle\langle VH| - |DD\rangle\langle DD| - |FF\rangle \times \langle FF| + |LL\rangle\langle LL| + |RR\rangle\langle RR|). \quad (15)$$

Using this technique for an unknown two-qubit state χ, the estimation of $\langle W\rangle$ requires three measurements [34], as is also evident from the decomposition of our witness operator for qubits in terms of Pauli spin matrices, i.e., $W = \frac{1}{4} \times [I \otimes I - \sigma_x \otimes \sigma_x + \sigma_y \otimes \sigma_y - \sigma_z \otimes \sigma_z]$, which is far less than the measurement of 15 parameters required for full state tomography [35]. In higher dimensions, the witness operator may be decomposed in terms of Gell-Mann matrices [17], and this difference further increases with the increase in dimensions. Therefore, the utility of the witness operator is indicated as compared to full state tomography when discrimination of useful entangled states for performing teleportation is required.

Before concluding, it may be noted that is possible to relate the FEF (1) with the maximum fidelity for other information processing tasks, such as super dense coding and entanglement swapping [36]. In the generalized dense coding for $d \otimes d$ systems, one can use a maximally entangled state $|\phi\rangle$ to encode $d^2/2$ bits in d^2 orthogonal states $(I \otimes U_i)|\phi\rangle$ [37]. If the maximally entangled state is replaced with a general density operator, the dense coding fidelity is defined as an average over the d^2 results. A relation between the maximum fidelity $F_{\mathrm{DC}}^{\max}$ of dense coding and the FEF was established for $d \otimes d$ systems to be $F_{\mathrm{DC}}^{\max} = F$. Similarly, for two-qubit systems the maximum fidelity of entanglement swapping [38] $F_{\mathrm{ES}}^{\max}$ is also related to the FEF by $F_{\mathrm{ES}}^{\max} = F$ [36]. However, teleportation is a different information processing task as compared to dense coding where $F > 1/d$ does not guarantee a higher than classical fidelity [39]. Hence, it is not possible to apply the above witness (7) to super dense coding and entanglement swapping.

Conclusions.—To summarize, in this work we have proposed a framework for discriminating quantum states useful for performing teleportation through the measurement of a Hermitian witness operator. The ability of an entangled state to act as a resource for teleportation is connected with the fully entangled fraction of the state. The estimation of the fully entangled fraction is difficult in general, except in the case of some known states. We have shown that the set of states having their fully entangled fraction bounded by a certain threshold value required for teleportation is both convex and compact. Exploiting this feature we have demonstrated the existence of a witness operator for teleportation. The measurement of the expectation value of the witness for unknown states reveals which states are useful as resource for performing teleportation. We have provided some illustrations of the applicability of the witness for isotropic and Werner states in $d \otimes d$ dimensions, and another class of maximally entangled mixed states for qubits. The measurability of such a witness operator requires determination of a much lesser number of parameters in comparison to state tomography of an unknown state, thus signifying the practical utility of our proposal. It would be interesting to explore the possibility of existence of witnesses for various other information processing tasks, as well. In this context further studies on finding optimal and common witnesses are called for.

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[1] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, England, 2000).
[2] C. H. Bennett and S. J. Wiesner, Phys. Rev. Lett. **69**, 2881 (1992).
[3] C. H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, and W. K. Wootters, Phys. Rev. Lett. **70**, 1895 (1993).
[4] A. K. Ekert, Phys. Rev. Lett. **67**, 661 (1991).
[5] M. Horodecki, P. Horodecki, and R. Horodecki, Phys. Lett. A **223**, 1 (1996).
[6] B. M. Terhal, Phys. Lett. A **271**, 319 (2000).
[7] R. B. Holmes, *Geometric Functional Analysis and its Applications* (Springer-Verlag, Berlin, 1975).
[8] W. Rudin, *Principles of Mathematical Analysis* (McGraw-Hill, New York, 1976).
[9] M. Lewenstein, B. Krauss, J. I. Cirac, and P. Horodecki, Phys. Rev. A **62**, 052310 (2000).
[10] B. M. Terhal, J. Theor. Comput. Sci. **287**, 313 (2002).
[11] J. Sperling and W. Vogel, Phys. Rev. A **79**, 022318 (2009).
[12] N. Ganguly and S. Adhikari, Phys. Rev. A **80**, 032331 (2009).
[13] O. Guhne and G. Toth, Phys. Rep. **474**, 1 (2009).
[14] B. M. Terhal and P. Horodecki, Phys. Rev. A **61**, 040301R (2000); A. Sanpera, D. Bruβ, and M. Lewenstein, Phys. Rev. A **63**, 050301R (2001).
[15] Y. C. Wu and G. C. Guo, Phys. Rev. A**75**, 052333 (2007); N. Ganguly, S. Adhikari, and A. S. Majumdar, arXiv:1101.0477.
[16] O. Guhne, P. Hyllus, D. Bruβ, A. Ekert, M. Lewenstein, C. Macchiavello, and A. Sanpera, Phys. Rev. A **66**, 062305 (2002).
[17] R. A. Bertlmann and P. Krammer, J. Phys. A **41**, 235303 (2008).
[18] C. Brukner and V. Vedral, arXiv:quant-ph/0406040; G. Toth, Phys. Rev. A **71**, 010301(R) (2005); J. Hide, W. Son, and V. Vedral, Phys. Rev. Lett. **102**, 100503 (2009).
[19] D. Bouwmeester, J.-W. Pan, K. Mattle, M. Eibl, H. Weinfurter, and A. Zeilinger, Nature (London) **390**, 575 (1997); S. Olmschenk, D. N. Matsukevich, P. Maunz, D. Hayes, L.-M. Duan, and C. Monroe, Science **323**, 486 (2009); X.-M. Jin *et al.*, Nature Photon. **4**, 376 (2010).
[20] R. F. Werner, Phys. Rev. A **40**, 4277 (1989).

[21] J. Lee and M. S. Kim, Phys. Rev. Lett. **84**, 4236 (2000).
[22] W. J. Munro, D. F. V. James, A. G. White, and P. G. Kwiat, Phys. Rev. A **64**, 030302 (2001).
[23] S. Adhikari, A. S. Majumdar, S. Roy, B. Ghosh, and N. Nayak, Quantum Inf. Comput. **10**, 0398 (2010).
[24] P. Horodecki, Phys. Lett. A **232**, 333 (1997); M. Horodecki, P. Horodecki, and R. Horodecki, Phys. Rev. Lett. **80**, 5239 (1998).
[25] C. H. Bennett, D. P. DiVincenzo, J. A. Smolin, and W. K. Wootters, Phys. Rev. A **54**, 3824 (1996).
[26] M. Horodecki, P. Horodecki, and R. Horodecki, Phys. Rev. A **60**, 1888 (1999).
[27] G. Vidal, D. Jonathan, and M. A. Nielsen, Phys. Rev. A **62**, 012304 (2000).
[28] M. J. Zhao, Z. G. Li, S. M. Fei, and Z. X. Wang, J. Phys. A **43**, 275203 (2010).
[29] R. J. Gu, M. Li, S. M. Fei, and X. Q. Li-Jost, Commun. Theor. Phys. **53**, 265 (2010).
[30] R. A. Bertlmann, K. Durstberger, B. C. Hiesmayr, and P. Krammer, Phys. Rev. A **72**, 052331 (2005).
[31] A. O. Pittenger and M. H. Rubin, Opt. Commun. **179**, 447 (2000); D. L. Deng and J. L. Chen, Ann. Phys. (Leipzig) **324**, 408 (2009).
[32] M. Ali, A. R. P. Rau, and G. Alber, Phys. Rev. A **81**, 042105 (2010).
[33] M. Barbieri, F. De Martini, G. Di Nepi, P. Mataloni, G. M. D'Adriano, and C. Macchiavello, Phys. Rev. Lett. **91**, 227901 (2003).
[34] P. Hyllus, Ph.D. thesis, University of Hannover, 2005.
[35] D. F. V. James, P. G. Kwiat, W. J. Munro, and A. G. White, Phys. Rev. A **64**, 052312 (2001).
[36] J. Grondalski, D. M. Etlinger, and D. F. V. James, Phys. Lett. A **300**, 573 (2002).
[37] X. S. Liu, G. L. Long, D. M. Tong, and F. Li, Phys. Rev. A **65**, 022304 (2002).
[38] M. Zukowski, A. Zeillinger, M. A. Horne, and A. K. Ekert, Phys. Rev. Lett. **71**, 4287 (1993).
[39] D. Bruβ, G. M. D'Ariano, M. Lewenstein, C. Macchiavello, A. Sen De, and U. Sen, Phys. Rev. Lett. **93**, 210501 (2004).