Effect of accretion on primordial black holes in Brans-Dicke theory

B. Nayak[*] and L. P. Singh[†]
Department of Physics, Utkal University, Vanivihar, Bhubaneswar 751004, India

A. S. Majumdar[‡]
S. N. Bose National Centre for Basic Sciences, Salt Lake, Kolkata 700098, India

We consider the effect of accretion of radiation in the early universe on primordial black holes in Brans-Dicke theory. The rate of growth of a primordial black hole due to accretion of radiation in Brans-Dicke theory is considerably smaller than the rate of growth of the cosmological horizon, thus making available sufficient radiation density for the black hole to accrete causally. We show that accretion of radiation by Brans-Dicke black holes overrides the effect of Hawking evaporation during the radiation dominated era. The subsequent evaporation of the black holes in later eras is further modified due to the variable gravitational "constant", and they could survive up to longer times compared to the case of standard cosmology. We estimate the impact of accretion on modification of the constraint on their initial mass fraction obtained from the γ-ray background limit from presently evaporating primordial black holes.

I. INTRODUCTION

The Brans-Dicke (BD) theory [1] which was proposed in 1961 is regarded as a viable alternative of Einstein's general theory of relativity (GTR). In the BD theory, the value of gravitational constant is set by the inverse of a time-dependent scalar field which couples to gravity with a coupling parameter ω. GTR can be recovered from this BD theory in the limit of $\omega \to \infty$. The BD theory has been used in attempts to understand many cosmological phenomena such as inflation [2], early and late time behaviour of the universe [3], cosmic acceleration and structure formation [4], and the coincidence problem[5]. It is also well known that BD-type models arise as low energy effective actions of several higher dimensional Kaluza-Klein and string theories [6].

Black holes which could be formed in the early Universe through a variety of mechanisms are known as primordial black holes [PBHs]. Some of the well-studied mechanisms of PBH formation include those due to inflation [7, 8], initial inhomogeneities [9, 10], phase transition and critical phenomena in gravitational collapse[11, 12], bubble collision [13] or the decay of cosmic loops [14] The formation masses of PBHs could be small enough for them to have evaporated completely by the present epoch due to Hawking evaporation [15]. Early evaporating PBHs could account for baryogenesis [16, 17] in the universe. On the other hand, longer lived PBHs could act as seeds for structure formation or even as precursors to supermassive black holes observed presently[18]. Furthermore, in certain scenarios it is possible for the PBHs to survive till date and form a significant component of dark matter [19]. It has been shown recently that PBHs in the braneworld scenario can efficiently accrete radiation [20, 21, 22] making them considerably long-lived.

The possibility of black hole solutions in BD theory was first proposed by Hawking [23]. Using scalar-tensor gravity theories Barrow and Carr [24] have studied PBH evaporation during various eras. It has been recently observed that in the context of generalised Brans-Dicke theory, inclusion of the effect of accretion leads to the prolongation of PBH lifetimes [25] . The coexistence of black holes with a long range scalar field in cosmology could have interesting consequences [26]. The possibility of variation of fundamental constants of nature over cosmological scales is a fascinating albeit contentious issue [27], and the black holes themselves could be used to constrain the variation of fundamental constants [28]. Another interesting issue of gravitational memory of black holes in BD theory has also been studied [29, 30].

Accretion of radiation by PBHs in the radiation dominated era of the early universe has been a much debated issue in standard cosmology. It is widely held that accretion is ineffective for sufficiently increasing the mass of a PBH [31], though later works have pointed out contrary possibilities in certain cases [17, 32, 33]. Recently, it has been realized in the context of the braneworld scenario that the possibility of enhanced accretion is quite favored due to the modified PBH geometry, as well as the modified early high energy era of the universe [20, 21]. Such a feature of effective early accretion prolonging the PBH lifetime by significant orders could also be valid for other modified gravity scenarios, as has already been shown in the context of a generalized scalar-tensor model [25].

The motivation for the present work is to study how accretion of radiation during the radiation dominated era impacts the evolution of primordial black holes in BD theory. Cosmological solutions during different eras in the BD theory were obtained by Barrow [34]. Here we consider together the processes of accretion of radiation

[*]bibeka@iopb.res.in
[†]lambodar_uu@yahoo.co.in
[‡]archan@bose.res.in

and Hawking evaporation for PBHs in BD theory using the solutions for the scale factor and the gravitational "constant" as used by Barrow and Carr [24]. In the present work we do not consider the effect of gravitational memory [29, 30] on PBH evolution, but assume that the evolution for the BD field is similar for the PBHs as it is for the whole universe. Cosmological observations could be used to impose constraints on the density of primordial black holes at various eras [35]. Within this formalism we also estimate the impact of accretion in modifying the constraint on their initial mass fraction in BD theory obtained from the γ-ray background limit from presently evaporating primordial black holes.

II. PBHS IN BRANS-DICKE THEORY

For a spatially flat($k = 0$) FRW universe with scale factor a , the Einstein equations and the equation of motion for the JBD field Φ take the form

$$\frac{\dot{a}^2}{a^2} + \frac{\dot{a}}{a}\frac{\dot{\Phi}}{\Phi} - \frac{\omega}{6}\frac{\dot{\Phi}^2}{\Phi^2} = \frac{8\pi\rho}{3\Phi} \tag{1}$$

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + 2\frac{\dot{a}}{a}\frac{\dot{\Phi}}{\Phi} + \frac{\omega}{2}\frac{\dot{\Phi}^2}{\Phi^2} + \frac{\ddot{\Phi}}{\Phi} = -\frac{8\pi p}{\Phi} \tag{2}$$

$$\frac{\ddot{\Phi}}{8\pi} + 3\frac{\dot{a}}{a}\frac{\dot{\Phi}}{8\pi} = \frac{\rho - 3p}{2\omega + 3}. \tag{3}$$

The energy conservation equation is given by

$$\dot{\rho} + 3(\gamma + 1)H\rho = 0 \tag{4}$$

assuming a perfect fluid equation of state $p = \gamma\rho$.

Barrow and Carr [24] have obtained the following solutions for a and G for different eras, as

$$a(t) \propto \begin{cases} t^{(1-\sqrt{n})/3} & (t < t_1) \\ t^{1/2} & (t_1 < t < t_e) \\ t^{(2-n)/3} & (t > t_e) \end{cases} \tag{5}$$

and

$$G(t) = \begin{cases} G_0\left(\frac{t_1}{t}\right)^{\sqrt{n}}\left(\frac{t_0}{t_e}\right)^n & (t < t_1) \\ G_0\left(\frac{t_0}{t_e}\right)^n & (t_1 < t < t_e) \\ G_0\left(\frac{t_0}{t}\right)^n & (t > t_e) \end{cases} \tag{6}$$

where $t_1 \sim$ is the time of starting of radiation dominated era, $t_e \sim$ is the era of matter-radiation equality, $t_0 \sim$ is the present time, $G_0 \sim$ is the present value of $G \simeq \frac{t_{pl}}{M_{pl}}$, and n is a parameter related to ω, i.e., $n = \frac{2}{4+3\omega}$. Since solar system observations [36] require that ω be large ($\omega \geq 10^4$), n is very small ($n \leq 0.00007$) .

Barrow and Carr have considered only evaporation of the primodial black holes due to Hawking radiation. If

we consider accretion which is effective in radiation dominated era, then the primordial black holes take more time to evaporate. Let us study how accretion changes the life time of the primodial black holes. For a primodial black hole immersed in the radiation field, the accretion of radiation leads to the increase of its mass with the rate given by

$$\dot{M}_{acc} = 4\pi f R_{BH}^2 \rho_R \tag{7}$$

where $R_{BH} = 2MG$ is the black hole radius, $\rho_R = \frac{3}{8\pi G}\left(\frac{\dot{a}}{a}\right)^2$ is the radiation energy density surrounding the black hole. and $f \sim$ is the accretion efficiency. The value of the efficiency of accretion f depends upon complex physical processes such as the mean free paths of the particles comprising the radiation surrounding the PBHs. Any peculiar velocity of the PBH with respect to the cosmic frame could increase the value of f [17]. Since the precise value of f is unknown, it is customary [21] to take the accretion rate to be proportional to the product of the surface area of the PBH and the energy density of radiation with $f \sim O(1)$. After substituting the above expressions for R_{BH} and ρ_R equation (7) becomes

$$\dot{M}_{acc} = 6fG\left(\frac{\dot{a}}{a}\right)^2 M^2 \tag{8}$$

Accretion is effective only in the radiation dominated era. So the primordial black holes which exist during the radiation dominated era are affected by accretion. Using the solutions for the scale factor $a(t)$ (5) and $G(t)$ (6) in equation (8), we get for the radiation dominated era

$$M(t) = \left[M_i^{-1} + \frac{3}{2}fG_0\left(\frac{t_0}{t_e}\right)^n\left(\frac{1}{t} - \frac{1}{t_i}\right)\right]^{-1} \tag{9}$$

where M_i is the black hole mass at it's formation time $t_i > t_1$. Assuming the standard mechanism for PBH formation due to gravitational collapse of density perturbations at the cosmological horizon scale [9], we have $M_i \simeq G^{-1}(t_i)t_i$. Since the horizon mass grows as $M_H(t) \sim G^{-1}t$, one finds that M_H grows faster than the black hole mass M_{BH} which for large times asymptotes to $M_i(1 - 3/2f)^{-1}$. Thus enough radiation density is available within the cosmological horizon for a PBH to accrete causally, making accretion effective in this scenario. However, the maximum accretion efficiency cannot exceed $f = 2/3$, thereby making it improbable for the overall mass of a PBH to increase much due to accretion. Nonetheless, the occurrence of accretion prolongs the onset of the evaporating era for the PBH thereby prolonging its lifetime considerably which in turn could significantly impact the observational constraints on PBHs in different eras.

Due to Hawking evaporation, the rate at which the primordial black hole mass decreases is given by

$$\dot{M}_{evap} = -4\pi R_{BH}^2 a_H T_{BH}^4 \tag{10}$$

where a_H is the Stefan-Boltzmann constant, and $T_{BH} = \frac{1}{8\pi GM}$ is the Hawking temperature. Now

$$\dot{M}_{evap} = -\frac{a_H}{256\pi^3}\frac{1}{G^2 M^2}. \qquad (11)$$

If we consider both evaporation and accretion simultaneously, then the rate at which the primordial black hole mass changes is given by

$$\dot{M}_{BH} = 6fG\left(\frac{\dot{a}}{a}\right)^2 M^2 - \frac{a_H}{256\pi^3}\frac{1}{G^2 M^2}. \qquad (12)$$

This equation can not be solved exactly. But we can very well approximate it during different regimes when either accretion or evaporation is the dominant process. Subsequently, we also integrate it by using numerical methods, which corroborates our approximation.

III.   PBH DYNAMICS IN DIFFERENT ERAS

A.   For $t < t_1$:

Hawking evaporation rate for this era is given by

$$\dot{M}_{evap} = -\alpha\left(\frac{t_e}{t_0}\right)^{2n}\left(\frac{1}{t_1}\right)^{2\sqrt{n}}\left(\frac{t^{2\sqrt{n}}}{M^2}\right) \qquad (13)$$

where $\alpha = \frac{a_H}{256\pi^3 G_0^2}$. Integrating the above equation, one gets

$$M^3 = M_i^3 + 3\alpha\left(1 + 2\sqrt{n}\right)^{-1}\left(\frac{t_e}{t_0}\right)^{2n}\left(t_1\right)^{-2\sqrt{n}} \\ \left(t_i^{2\sqrt{n}+1} - t^{2\sqrt{n}+1}\right) \qquad (14)$$

This regime corresponds to the BD field dominated dynamics, where the radiation density is only subdominant. Assuming accretion is not effective in this era, we get same result as that of Barrow and Carr [24] for the evaporation time $(M = 0)$.

$$\tau = \left[(3\alpha)^{-1}(1 + 2\sqrt{n})\left(\frac{t_0}{t_e}\right)^{2n}M_i^3 t_1^{2\sqrt{n}} + t_i^{1+2\sqrt{n}}\right]^{\frac{1}{1+2\sqrt{n}}} \qquad (15)$$

B.   For $t_1 < t < t_e$:

This period corresponds to the radiation dominated era. Here we consider two possibilities: PBHs created before t_1 and those created after t_1.

CASE-I $(t_i < t_1)$:
The PBH evaporation equation (11) becomes

$$\int_{M_i}^{M} M^2\, dM = -\alpha\Big[\int_{t_i}^{t_1}\left(\frac{1}{t_1}\right)^{2\sqrt{n}}\left(\frac{t_e}{t_0}\right)^{2n}t^{2\sqrt{n}}\, dt \\ + \int_{t_1}^{t}\left(\frac{t_e}{t_0}\right)^{2n} dt\Big]. \qquad (16)$$

Integrating this equation, one gets

$$M^3 = M_i^3 + 3\alpha\left(\frac{t_e}{t_0}\right)^{2n}\Big[t_1 - t \\ + \left(\frac{1}{t_1}\right)^{2\sqrt{n}}\left(1 + 2\sqrt{n}\right)^{-1}\left(t_i^{2\sqrt{n}+1} - t_1^{2\sqrt{n}+1}\right)\Big]. \qquad (17)$$

Now considering both evaporation and accretion, we obtain

$$\dot{M}_{BH} = -\alpha\left(\frac{1}{t_1}\right)^{2\sqrt{n}}\left(\frac{t_e}{t_0}\right)^{2n}\left(\frac{t^{2\sqrt{n}}}{M^2}\right) \qquad (18)$$

during the BD field dominated era $(t < t_1)$ and

$$\dot{M}_{BH} = \frac{3}{2}fG_0\left(\frac{t_0}{t_e}\right)^{n}\left(\frac{M^2}{t^2}\right) - \alpha\left(\frac{t_e}{t_0}\right)^{2n}\frac{1}{M^2} \qquad (19)$$

for $t >> t_1$ in the radiation dominated era. Since accretion is effective only during the radiation dominated era, Eq.(18) is first integrated over the time period t_i to t_1 and then Eq.(19) is then integrated over the time period t_1 to t with the initial condition $M_{BH}(t_1)$ obtained from the solution of Eq.(18). The results of numerical integration of the above equation with several values of M_i are presented in the figure-1. We assume that radiation domination sets in at the GUT scale, i.e., $t_1 = 10^{-35}s$. We find that the PBHs formed before the onset of radiation domination, i.e., for $t_i < t_1$, evaporate out completely during the radiation dominated era. Figure 1 shows the variation of PBH masses for three different values of M_i with $M_i \sim 990g$ corresponding to the formation time $t \sim t_1$. Though here the accretion efficiency $f = 1/3$, it can be verified that even for larger f, accretion in this case is not able to prolong PBH lifetimes beyond the radiation dominated era, i.e., those PBHs which form at a time $t_i < t_1$, have evaporations times $t_{evap} < t_e$. Note that though in our analysis we assume radiation domination at the GUT scale, it is possible for the radiation domination to set in much later, i.e., $t_1 >> 10^{-35}s$ in certain scenarios, e.g., gravitino production could constrain the reheating temperature to be much lower [37]. In such a case of low reheating temperature, accretion of radiation will be effective for a shorter window of time corresponding to a shortened span of radiation domination, and the evolution of PBHs would be closer to the case in BD theory without accretion [24].

CASE-II $(t_i > t_1)$:

Taking both accretion and evaporation into account, we can write

$$\dot{M}_{BH} = \frac{3}{2}fG_0\left(\frac{t_0}{t_e}\right)^{n}\left(\frac{M^2}{t^2}\right) - \alpha\left(\frac{t_e}{t_0}\right)^{2n}\frac{1}{M^2}. \qquad (20)$$

For PBHs with formation mass $M_i^2 > \frac{a_H G^{-1}}{384f}$, the magnitude of the first term (accretion) exceeds that of the second term (evaporation). In the radiation dominated era for a PBH whose formation mass satisfies the above
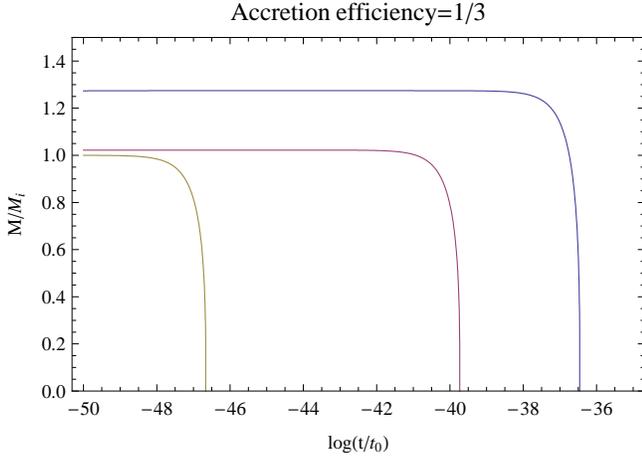
FIG. 1: Evolution of PBH masses with time (t_0 being the present age of the universe) for different initial mass $M_i = 0.5g, 100g, 990g$, but with same accretion efficiency $f = 1/3$.

relation, accretion is dominant upto a value of t, say t_c at which accretion equals evaporation (the PBH mass rises to a maximum value M_{max} at this stage), and after that evaporation dominates over accretion. For $t = t_c$, the magnitude of the accretion term is equal to the magnitude of evaporation term. So for the radiation dominated era, equation (20) implies,

$$\frac{3}{2}fG_0\left(\frac{t_0}{t_e}\right)^n\left(\frac{M_{max}^2}{t^2}\right) = \alpha\left(\frac{t_e}{t_0}\right)^{2n}\left(\frac{1}{M_{max}^2}\right) \quad (21)$$

which gives

$$M_{max} = \left(\frac{A}{f}\right)^{\frac{1}{4}} \times \left(t_c\right)^{\frac{1}{2}} \quad (22)$$

where $A = \frac{2}{3}G_0^{-1}\alpha\left(\frac{t_e}{t_0}\right)^{3n}$ and $M_{max} = M(t_c)$. But from the PBH accretion equation (9), we have

$$M_{max} = M_i\left[1 + \frac{3}{2}f\left(\frac{t_i}{t_c} - 1\right)\right]^{-1}. \quad (23)$$

Equating above two expressions for M_{max}, one gets

$$t_c^{1/2} = \left(\frac{f}{A}\right)^{\frac{1}{4}} \times \frac{M_i}{1 - \frac{3}{2}f} \quad (24)$$

and

$$M_{max} = \frac{M_i}{1 - \frac{3}{2}f} \quad (25)$$

which again stipulates that $f < \frac{2}{3}$.

Considering evaporation from t_c onwards, we get

$$M = M_{max}\left[1 + 3\alpha\left(\frac{t_e}{t_0}\right)^{2n}\left(\frac{t_c}{M_{max}^3}\right)\left\{1 - \left(\frac{t}{t_c}\right)\right\}\right]^{\frac{1}{3}}. \quad (26)$$

So the evaporation time for these PBHs are given by

$$t_{evap} = t_c\left[1 + \left(3\alpha\right)^{-1}\left(\frac{t_0}{t_e}\right)^{2n}\left(\frac{M_{max}^3}{t_c}\right)\right]. \quad (27)$$

C. For $t > t_e$:

The PBHs which are formed before radiation domination completely evaporate out during the radiation dominated era. So for $t > t_e$, only those PBHs which are formed after t_1 exist. The PBH evaporation equation (11) can be written as

$$\int_{M_i}^{M} M^2\, dM = -\alpha\left[\int_{t_i}^{t_e}\left(\frac{t_e}{t_0}\right)^{2n} dt + \int_{t_e}^{t}\left(t_0\right)^{-2n}t^{2n}\, dt\right]. \quad (28)$$

Taking both accretion and evaporation into account, one gets

$$\dot{M}_{BH} = \frac{3}{2}fG_0\left(\frac{t_0}{t_e}\right)^n\left(\frac{M^2}{t^2}\right) - \alpha\left(\frac{t_e}{t_0}\right)^{2n}\frac{1}{M^2} \quad (29)$$

during the radiation dominated era and

$$\dot{M}_{BH} = -\alpha\left(\frac{1}{t_0}\right)^{2n}\left(\frac{t^{2n}}{M^2}\right) \quad (30)$$

during the matter dominated era. In order to obtain the PBH lifetime, one can numerically integrate the above equations (29) and (30). Eq.(29) corresponding to accretion and evaporation during the radiation dominated era is integrated over the period t_i to t_e and the value of $M_{BH}(t_e)$ is obtained and used as the initial condition for integrating Eq.(30) over the period t_e to t. The results of numerical integration are displayed for a particular initial mass in figure-2. (In the figure we do not show the early part of their evolution where their mass increases by a bit due to accretion and then stays nearly constant for a long period of time). One sees that depending on the accretion efficiency f, the lifetimes of the PBHs formed during the radiation dominated era could exceed the present age of the universe t_0.
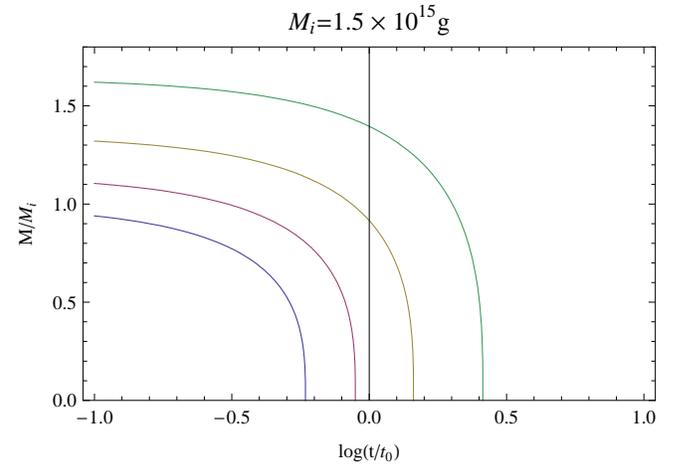

FIG. 2: The late time evolution of PBH masses (with the same initial mass $M_i = 1.5 \times 10^{15}g$) for various accretion efficiency values $f = 0, 0.2, 0.4, 0.6$.

On the other hand, integrating the last term in equation (29) one can obtain the mass of the PBHs which

survive beyond the radiation dominated era, given by

$$M = M_e\Big[1 + 3\alpha\Big(2n+1\Big)^{-1}\Big(\frac{t_e}{t_0}\Big)^{2n}\Big(\frac{t_e}{M_e^3}\Big)$$
$$\Big\{1 - \Big(\frac{t}{t_e}\Big)^{2n+1}\Big\}\Big]^{\frac{1}{3}} \qquad (31)$$

where $M_e \equiv M(t_e)$ is obtained by integration of the PBH accretion equation (8) over the period t_i to t_e to be

$$M_e = M_i\Big[1 + \frac{3}{2}fG_0M_i\Big(\frac{t_0}{t_e}\Big)^n\Big(\frac{1}{t_i} - \frac{1}{t_e}\Big)\Big]. \qquad (32)$$

Hence, the PBH lifetime is given by

$$t_{evap} = t_e\Big[1 + \Big(3\alpha\Big)^{-1}(2n+1)\Big(\frac{t_0}{t_e}\Big)^{2n}\Big(\frac{M_e^3}{t_e}\Big)\Big]^{\frac{1}{2n+1}}. \qquad (33)$$

Further, from equation (26), we have

$$M_e = M_{max}\Big[1 + 3\alpha\Big(\frac{t_e}{t_0}\Big)^{2n}\Big(\frac{t_c}{M_{max}^3}\Big)$$
$$\Big\{1 - \Big(\frac{t_e}{t_c}\Big)\Big\}\Big]^{\frac{1}{3}}. \qquad (34)$$

Now using the equations (33) and (25), we get

$$M_i \approx \Big\{1 - \frac{3}{2}f\Big\} \times \Big[3\alpha\Big(\frac{t_e}{t_0}\Big)^{2n}t_e\Big\{1 + \Big(2n+1\Big)^{-1}$$
$$\Big\{\Big(\frac{t_{evap}}{t_e}\Big)^{2n+1} - 1\Big\}\Big\}\Big]^{\frac{1}{3}}. \qquad (35)$$

This enables us to invert the PBH lifetime relation in order to obtain the formation time for a PBH given its time of evaporation t_{evap},

$$t_i \approx G_0\Big\{1 - \frac{3}{2}f\Big\}\Big(\frac{t_0}{t_e}\Big)^n \times \Big[3\alpha\Big(\frac{t_e}{t_0}\Big)^{2n}t_e$$
$$\Big\{1 + \Big(2n+1\Big)^{-1}\Big(\frac{t_{evap}}{t_e}\Big)^{2n+1}\Big\}\Big]^{\frac{1}{3}}. \qquad (36)$$

The above expression is useful for the purpose of evaluating the constraints on the initial mass or formation time of the PBHs in terms of the observational constraints on evaporating black holes at particular eras in this Brans-Dicke cosmology. For the present we compute as examples the initial masses of the PBHS for two cases: (i) PBHs that are evaporating at the present era, and (ii) the PBHs that will evaporate when the universe is ten times older than its present age. These values are computed using the analytical result (36) and displayed in the Table I. We have also computed t_i and M_i for different values of the accretion efficiency f from numerical integration of equation (29). We find that our analytical approach gives results that agree up to three decimal places with the numerical results, thus validating the division of the evolution of a PBH into two distinct eras dominated by accretion and evaporation dynamics respectively, that we have done.

	$t_{evap} = t_0$		$t_{evap} = 10 \times t_0$	
f	$t_i \times 10^{-23}$s	$M_i \times 10^{15}$g	$t_i \times 10^{-23}$s	$M_i \times 10^{15}$g
0	2.369	2.366	5.105	5.099
0.2	1.658	1.656	3.573	3.569
0.4	0.947	0.946	2.042	2.039
0.6	0.236	0.236	0.510	0.509

TABLE I: The formation times and initial masses corresponding to two specific evaporating eras of the PBHs are displayed for several accretion efficiencies.

IV. CONSTRAINTS ON THE PBH MASS FRACTION

Surviving PBHs at any cosmological era contribute to the matter density of the universe at that era. Further, PBHs impact different processes by the end products of their Hawking evaporation. Various cosmological observations can be used to impose constraints on the number density of black holes present during different cosmological eras. These constraints can in turn be used for imposing limits of the initial mass spectrum of PBHs in various formation mechanisms pertaining to different cosmological models. In standard cosmology, a variety of constraints such as coming from considerations of overall density, nuleosynthesis, entropy, distortions of the CMBR spectrum, and stable relics, have been obtained [35]. It has been observed that a particular stringent set of constraints arise from the limits of the γ-ray background [38], and also independently from the observed galactic anti-protons and antideuterons [39]. In the present analysis we will just focus on the γ-ray background limit in order to obtain bounds on the initial mass spectrum of PBHs in BD theory with accretion.

The fraction of the Universe's mass going into PBHs at time t is given by [9]

$$\beta(t) = \Big[\frac{\Omega_{PBH}(t)}{\Omega_R}\Big](1+z)^{-1} \qquad (37)$$

where $\Omega_{PBH}(t)$ is the present density parameter associated with PBHs forming at time t, z is the redshift associated with time t and Ω_R is the present microwave background density having value 10^{-4}. Observations of the γ-ray background, as well as those of the antiprotons from galactic sources impose bounds on the present cosmological PBH density given by [24, 38, 39]

$$\Omega_{PBH}(t) < 10^{-8} \qquad (38)$$

Let us here consider the PBHs that are formed during the radiation dominated era, i.e., $t_1 < t_i < t_e$. For them, one has

$$(1+z)^{-1} = \Big(\frac{t}{t_e}\Big)^{\frac{1}{2}}\Big(\frac{t_e}{t_0}\Big)^{\frac{2-n}{3}}. \qquad (39)$$

Using equations (38) and (39) in equation (37), one gets

$t_{evap} = t_0$		
f	$M_i \times 10^{15}\text{g}$	$\beta(M_i) <$
0	2.366	5.71×10^{-26}
1/6	1.775	4.95×10^{-26}
1/3	1.183	4.03×10^{-26}
1/2	0.592	2.85×10^{-26}
3/5	0.236	1.81×10^{-26}

TABLE II: Upper bounds on the initial mass fraction of PBHs that are evaporating today for various accretion efficiencies f.

a bound on the density fraction given by

$$\beta(t) < 10^{-4} \times \left(\frac{t}{t_e}\right)^{\frac{1}{2}} \times \left(\frac{t_e}{t_0}\right)^{\frac{2-n}{3}}. \qquad (40)$$

The above bound pertains to those PBHs which are evaporating at the present era. Again, for $t_1 < t < t_e$, one has $M = G^{-1}t = G_0^{-1}t\left(\frac{t_e}{t_0}\right)^n$. Thus, the fraction of the Universe going into PBHs with formation mass M_i is

$$\beta(M_i) < 10^{-4} \times \left(\frac{M_i}{M_e}\right)^{\frac{1}{2}} \times \left(\frac{t_e}{t_0}\right)^{\frac{2-n}{3}}. \qquad (41)$$

We can now use the expressions for M_i (35) in terms of the evaporation time $t_{evap} = t_0$ to obtain the values of $\beta(M_i)$ corresponding to various values of the accretion efficiency f. These are displayed in Table II. It was shown earlier [38] how the standard constraints on the initial mass spectrum are modified in BD theory without accretion. Here we observe that increase of accretion efficiency makes the limits on the initial mass fraction more stringent. It may be noted that similar considerations would also apply to constraints on the initial mass fraction $\beta(M_i)$ obtained from other physical considerations such as those due to entropy or nucleosynthesis bounds. The relevant values for the initial PBH masses M_i corresponding to the PBHs evaporating earlier to impact entropy production or nucleosynthesis would of course be much lower than the values of M_i for which we have applied the γ-ray bounds, since these latter PBHs are those that are evaporating in the present era. The BD dynamics alters the evaporation rate for the PBHs thus loosening somewhat the bounds on $\beta(M_i)$ as shown by Barrow and Carr [24]. However, inclusion of accretion reverses the scenario since accretion is more effective for a longer duration for PBHs with smaller M_i which have a chance to grow more. As a consequence, $\Omega_{PBH}(t)$ increases, and the standard constraints on $\beta(M_i)$ due to nucleosynthesis, entropy, etc., are tightened further in the BD scenario with accretion.

V. CONCLUSIONS

In this paper we have considered the evolution of primordial black holes in Brans-Dicke theory. We use the framework of a particular cosmological solution [34] where the gravitational "constant" G can have a much larger value in the early universe compared to its present strength. We show that accretion of radiation during the radiation dominated era is an effective process in this theory, overriding the mass loss of the PBHs due to Hawking evaporation. The cosmological horizon for a PBH grows at a faster rate compared to the rate of growth of the PBH due to accretion, thus making enough radiation available for the PBH to accrete during this stage. Though the net gain of mass through accretion is not significant, it postpones the time for evaporation to take over once accretion ceases to be effective at the onset of the matter dominated era. The evaporation rate for a BD PBH is itself modifed compared to that in standard cosmology due the variable G. We show that the lifetime for PBHs could be enhanced depending upon the accretion efficiency f, making the PBHs that are supposed to be evaporating now (t_0) live longer by a factor of $\left[\frac{1}{1-\frac{3}{2}f}\right]^{3/(2n+1)} \times t_0$.

The cosmological evolution of PBHs could lead to various interesting consequences during different eras. The PBHs with smaller masses that are formed during the BD field dominated very early era evaporate out completely during the radiation dominated era. However, those PBHs that are formed later during the radiation dominated era survive much longer. These are the ones that through their evaporation could impact various cosmological processes such as nucleosynthesis and photon decoupling. There exists a variety of observational constraints on the PBHs in standard cosmology [35]. Within the context of BD theory Barrow and Carr [24] have evaluated the impact of density bounds on the PBHs evaporating today on their initial mass spectrum. Here we use the observational limits on the γ-ray background [38] to compute the effect of accretion on constraining the initial mass fraction of the PBHs. The departure of these constraints from those in standard cosmology are quite sensitive to the accretion efficiency.

Finally, it may be noted that there exist other interesting cosmological solutions for the Brans-Dicke theory and its extensions to more general scalar-tensor models [40]. In a recent paper one such solution [3] with a time-evolving BD coupling parameter ω was used to study cosmological PBH evolution [25]. It could be worthwhile to remap the standard observational constraints on PBHs in such models in order to estimate the impact of rapidly varying G, as well as possible effective accretion on the constraints. Another important issue in PBH evolution could be the impact of the back reaction of the PBHs on the local background value of the BD field resulting from the local change of the density due to the PBHs. Back reaction could indeed lead to non-trivial consequences on cosmological evolution [41], and in the present context it might be interesting to see in what way any resulting modification could in turn impact the evolution of black holes in Brans-Dicke theory.

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