Astrophysical constraints on primordial black holes in Brans-Dicke theory

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We consider cosmological evolution in Brans-Dicke theory with a population of primordial black holes. Hawking radiation from the primordial black holes impacts various astrophysical processes during the evolution of the Universe. The accretion of radiation by the black holes in the radiation dominated era may be effective in imparting them a longer lifetime. We present a detailed study of how this affects various standard astrophysical constraints coming from the evaporation of primordial black holes. We analyze constraints arising from the present density of the Universe, the present photon spectrum, the distortion of the cosmic microwave background spectrum and also from processes affecting light element abundances after nucleosynthesis. We find that the constraints on the initial primordial black hole mass fractions are tightened with increased accretion efficiency.

Keywords: primordial black holes, modified gravity

Contents

I.   INTRODUCTION

The Standard Model of Cosmology invokes General Theory of Relativity(GTR) as the theory of gravity. However, all the confirmative tests of GTR have been carried out at low energy. This has led people to believe and explore the deviations from GTR at high energy regimes like cosmic evolution at very early times. Brans-Dicke (BD) theory of gravitation [1] stands out as one of the most attractive alternatives to GTR because it involves minimal extension over GTR through introduction of a scalar field ϕ. In the BD theory the gravitational constant becomes function of time and is proportional to the inverse of the scalar field ϕ which couples to gravity with a coupling parameter ω. The more general nature of BD theory is evident from the fact that in the limit $\omega \to \infty$ it goes over to GTR. Solar system observations require $\omega > 10^4$ [2]. The ubiquitous nature of BD theory is also evident from the fact that it appears in the low energy limit of Kaluza-Klein and String theories [3]. Thus, BD theory has been used for tackling

a number of cosmological problems such as inflation [4], early and late time behaviour of the Universe [5], cosmic acceleration and structure formation [6], coincidence problem [7] and problems relating to black holes [8].

It has been pointed out that in the early stage of the Universe Primordial Black Holes (PBHs) could be formed due to various mechanisms such as inflation [9], initial inhomogeneous conditions [10, 11], phase transition [12], bubble collision [13] and decay of cosmic loops [14]. In the usual formation scenarios, the typical mass of PBHs at the formation time could be as large as the mass contained in the Hubble volume M_H ranging down to about $10^{-4}M_H$ [15]. The formation masses of PBHs could be thus small enough to have evaporated completely by the present epoch due to Hawking evaporation [16]. However, since the cosmological environment is very hot and dense in the radiation-dominated era, it is expected that appreciable absorption of the energy-matter from the surroundings could take place. It has been noticed that such accretion is most effective in altered gravity scenarios where the PBHs grow due to accretion of radiation at a rate smaller than that of the Hubble volume, thus providing for enough energy density for the PBHs to accrete causally. This is responsible for the prolongation of the lifetime of PBHs in braneworld models [17] as well as in scalar-tensor models [18].

The feasibility of black hole solutions in BD theory was first discussed by Hawking [19]. Using scalar-tensor gravity theories Barrow and Carr [20] have studied PBH evaporation during various eras. It has been recently observed [21] that in the context of Brans-Dicke theory, inclusion of accretion leads to the prolongation of PBH lifetime. Once formed, these PBHs will influence later cosmological epochs, leading to a number of observational constraints on their allowed abundance. These have been extensively investigated in the case of standard cosmology [22–24]. In a recent work, Carr et al. [25] have performed a detailed numerical study of the effect of the emission of quark and gluons by PBHs on the standard constraints. The standard constraints also get altered in different gravitational theories, as was studied in the context of Brans-Dicke cosmology without accretion [20], and brane-world cosmology [26–28]. The aim of the present paper is to reanalyse the main constraints in the Brans-Dicke theory with the inclusion of accretion of radiation in the early Universe. The standard constraint formalism could get modified due to the change in theory of gravity and also due to the effect of accretion at early times, as was shown in the case of braneworld gravity by Clancy *et al.* [26]. In the present study we extend our previous work on the evolution of PBHs in BD theory including accretion [21] to obtain several astrophysical constraints following the style of Clancy *et al.* [26].

The plan of the paper is as follows. In the next section we first provide the key expressions related to PBH evaporation, accretion and lifetimes in the BD scenario, and then discuss the modified constraint formalism in BD theory that we use subsequently. In Section III we consider the observational constraints obtained from the present density of the Universe. In Section IV we discuss the present photon spectrum and constraints following from it. The constraints arising out of the distortion of the Cosmic Microwave Background (CMB) spectrum are evaluated in Section V. Light element abundances and photo-disintegration of the deuterium nuclei lead to further constraints on the initial PBH mass fraction, which are presented in Section VI. A summary of our results along with a Table with quantitative estimates of the various constraints are presented in Section VII.

II. THE CONSTRAINT FORMALISM IN BRANS-DICKE THEORY

For discussing the constraints that can arise from PBH evaporation, we label an epoch by cosmic time t. PBHs which are not evaporated by time t will only contribute to the overall energy density of PBHs. As the present observable Universe is nearly flat and, therefore, possesses critical density, the PBH mass density can be constrained on the ground that it should not overdominate the Universe. PBHs evaporate by producing bursts of evaporation products. The Hawking radiation from the PBHs which evaporate well before photon decoupling will thermalize with the surroundings, boosting the photon-to-baryon ratio [29]. In the case of evaporation after photon decoupling, the radiation spectrum is affected and subsequently redshifts in a monotonic manner. Thus, constraints arise from the cosmic background radiation at high frequencies [30, 34, 41]. Further, if the PBHs evaporate close to the time of photon decoupling, it cannot be fully thermalised and will produce distortion in the cosmic microwave background spectrum. Generally speaking, at a given epoch, the constraint on various physical observables is usually dominated by those

PBHs with a lifetime of order of the epoch in question. Hence, the observational constraint can be translated into an upper limit on the initial mass fraction of PBHs.

We consider a flat FRW Universe which is radiation dominated upto a time t_e and matter-dominated thereafter. In the radiation-dominated era [20]

$$a(t) \propto t^{1/2}; \quad G = G_0 \left(\frac{t_0}{t_e}\right)^n, \tag{1}$$

where G_0 denotes the present value of G, and n is related to the Brans-Dicke parameter w by $n = 2/(4+3w)$. [In view of the observational bound on w [2], we consider the value of n as $n \sim 0.0001$ in our subsequent calculations]. The radius and temperature of a PBH is given by

$$r = 2G_0 \left(\frac{t_0}{t_e}\right)^n M; \quad T_{BH} = \frac{m_{pl}^2}{8\pi M} \left(\frac{t_e}{t_0}\right)^n. \tag{2}$$

Similarly, in the matter-dominated era

$$a(t) \propto t^{(2-n)/3}; \quad G(t) = G_0 \left(\frac{t_0}{t}\right)^n, \tag{3}$$

and

$$r = 2G_0 \left(\frac{t_0}{t}\right)^n M \quad and \quad T_{BH} = \frac{m_{pl}^2}{8\pi M} \left(\frac{t}{t_0}\right)^n. \tag{4}$$

In Brans-Dicke theory the evolution of a PBH [21] is described by

$$\frac{dM}{dt} = \left(\frac{dM}{dt}\right)_{acc} + \left(\frac{dM}{dt}\right)_{evap} \tag{5}$$

with the accretion and evaporation rates given by

$$\left(\frac{dM}{dt}\right)_{acc} = 6fG_0 \left(\frac{t_0}{t_e}\right)^n \left(\frac{\dot{a}}{a}\right)^2 M^2 \tag{6}$$

and

$$\left(\frac{dM}{dt}\right)_{evap} = -\alpha \left(\frac{t_e}{t_0}\right)^{2n} \frac{1}{M^2} \tag{7}$$

where $\alpha = \sigma/(256\pi^3 G_0^2)$, with σ the Stefan-Boltzmann constant. f is the accretion efficiency which depends primarily on two factors: (i) how small the size of the black hole is compared to the cosmological horizon, and (ii) how efficient the black hole is in absorbing background radiation. The first factor is contingent upon various formation mechanisms. The second factor could depend, in turn on the thermal properties of the surrounding radiation, the mean free path of absorbed particles, as well as the backreaction due the black hole on the cosmological metric. Due to several uncertainties in the estimation of the above quantities, it is usually accepted to consider an accretion efficiency f in the range between 0 and 1.

Accretion makes the mass of PBH to grow a maximum value [21]

$$M_{max} = M(t_c) = \frac{M_i}{1 - \frac{3}{2}f} \tag{8}$$

where M_i is the initial mass of PBH and t_c is the time at which evaporation rate is equal to the accretion rate. Thus f is further restricted to lie in between 0 and $\frac{2}{3}$ in Brans-Dicke theory. We assume the standard mechanism of PBH formation due to the gravitational collapse of density perturbations at the cosmological horizon scale [11], which leads to $M_i \simeq G^{-1}(t_i)t_i$.

The evaporation time of PBHs which completely evaporate in the radiation dominated era, is given by [21]

$$t_{evap} = t_c \left[1 + (3\alpha)^{-1} \left(\frac{t_0}{t_e} \right)^{2n} \left(\frac{M(t_c)^3}{t_c} \right) \right].$$

(9)

The PBHs which evaporate in the matter dominated era have their initial mass and evaporation time related as [21]

$$M_i = \left\{ 1 - \frac{3}{2}f \right\} \times \left[3\alpha \left(\frac{t_e}{t_0} \right)^{2n} t_e \left\{ 1 + (2n+1)^{-1} \times \left\{ \left(\frac{t_{evap}}{t_e} \right)^{2n+1} - 1 \right\} \right\} \right]^{1/3}.$$

(10)

Constraints on the allowed abundance of PBHs of a certain lifetime are formulated as upper bounds on their mass fraction. This mass fraction $\alpha_t(M_i)$ is defined as the ratio of the energy density due to PBHs of initial mass M_i and the background radiation density, at a time $t \geq t_i$, as

$$\alpha_t(M_i) = \frac{\rho_{PBH,M_i}(t)}{\rho_{rad}(t)}.$$

(11)

The black holes first gain mass through accretion up to time t_c. During this stage α_t scales as (Ma). Subsequently, they begin to loose mass very gradually (i.e., the mass stays nearly constant, and therefore α_t scales as a). This stage comes to an end when α_t reaches the value α_{evap} beyond which the black hole looses most of its mass in a final burst of evaporation. It then follows that if initial and final mass fractions denoted by α_i and α_{evap} respectively, are related by

$$\alpha_{evap} = \alpha_i \frac{M(t_c)}{M(t_i)} \frac{a(t_{evap})}{a(t_i)}.$$

(12)

Using equation (8), we can write

$$\alpha_{evap} = \alpha_i \left(\frac{1}{1 - \frac{3}{2}f} \right) \frac{a(t_{evap})}{a(t_i)}.$$

(13)

The purpose of the following sections is to reconsider observational constraints on α_{evap} at different cosmological epochs, trace them back to obtain constraints on the initial mass fraction α_i taking into consideration accretion of radiation in the early universe by the PBHs in Brans-Dicke theory. It may be mentioned here that the recent paper by Carr et al. [25] has reanalyzed the standard constraints on primordial black holes by considering the effects of emission of quarks and gluons and the resultant secondary emission of photons. It was shown that the effect of secondary photon emission could alter the standard constraints on PBH fraction by a couple of orders of magnitude in certain cases, e.g., deuterium constraint, while leaving the standard constraints more or less unaltered in certain other cases, e.g., distortion of CMB spectrum. The results obtained by Carr et al. [25] are based on detailed numerical analysis. Of course, such a scenario of emission would also impact constraints on Brans-Dicke primordial black holes in more or less similar ways as they impact PBHs in the standard picture. However, the lack of analytical results describing the effect of such emission on the constraint formalism makes it considerably harder to perform a similar analysis in the context of an altered gravitational scenario. Since the primary aim of the present paper is to study the effect of Brans-Dicke theory on the constraint formalism, we do not attempt to perform any similar numerical analysis here. In stead, we make a note of the effects that emission of secondary photons could have on the different constraints, based on the results of Ref. [25].

III. THE PRESENT MATTER DENSITY OF THE UNIVERSE

For a particular value of f in Eq.(8) with $t_{evap} = t_0$, and using Eq.(10) one gets the formation time of PBHs that are evaporating today. Black holes formed later are essentially still intact and their density is constrained by the observed matter density in the present Universe. Here we are tracking the relative

densities of PBHs to radiation and we must ensure that, given the observed radiation density, this ratio does not imply that the PBH density exceeds the observed matter density of about 0.3 of the critical density. Phrased in this way, the constraint applies regardless of the presence of a cosmological constant and indicates that for any PBHs surviving to the present we must have

$$\alpha_0(M) < \frac{0.3}{\Omega_{\gamma,0}}. \tag{14}$$

The cosmic microwave background corresponds to a photon density of $\Omega_{\gamma,0}h^2 = 2.47 \times 10^{-5}$ (with $h = 0.7$) and conservatively, we can ignore the cosmic neutrinos. Thus, for PBHs that are about to evaporate today, $t_{evap} \geq t_0$, we get

$$\alpha_{evap} < \frac{0.3}{\Omega_{\gamma,0}} \approx 6 \times 10^3. \tag{15}$$

Using equation(13), one can find

$$\alpha_i < 3.43 \times 10^{-18} \times \left(1 - \frac{3}{2}f\right)^{3/2}. \tag{16}$$

The constraint on the initial PBH mass fraction in the tandard cosmology is obtained as [31]

$$\alpha_i < 10^{-18}. \tag{17}$$

IV. THE PRESENT PHOTON SPECTRUM

If PBHs evaporate between the time of photon decoupling(t_{dec}) and the present day, their radiation spectra will not be appreciably influenced by the background Universe, apart from being redshifted. Thus the spectra could constitute a fraction of the cosmic background radiation. The number of particles of a certain species j, emitted in 4-dimensional spacetime by a black hole of temperature T_{BH}, in a time interval dt and with momentum in the interval (k, k+dk) is

$$dN_j = \sigma_j(k)\frac{dt}{exp(\frac{\omega}{T_{BH}}) \pm 1}\frac{d^3k}{(2\pi)^3} \tag{18}$$

with $\omega^2 = k^2 + m^2$, where $\sigma_j(k)$ is the emission cross-section for species j with momentum k and m is the mass of the particle.

For emission of photons with energy in the interval (E, E+dE) above equation becomes

$$dN = \frac{\sigma(E)}{2\pi^2}\frac{E^2}{exp(\frac{E}{T_{BH}}) - 1}dEdt. \tag{19}$$

Thus the spectral photon number emitted by a small black hole with lifetime t_{evap} is obtained from

$$\frac{dN}{dE} = \int_0^{t_{evap}} \frac{\sigma(E)}{2\pi^2}\frac{E^2}{exp(\frac{E}{T_{BH}}) - 1} dt. \tag{20}$$

But, in the high frequency limit $E >> T_{BH}$, all cross section reduce to the same value [26] as

$$\sigma = 4\pi r_0^2 = \frac{1}{4\pi T_{BH}^2}. \tag{21}$$

Now, the spectral number becomes

$$\frac{dN}{dE} = \frac{1}{8\pi^3} \int_0^{t_{evap}} \frac{(\frac{E}{T_{BH}})^2}{exp(\frac{E}{T_{BH}}) - 1} dt. \tag{22}$$

In the high energy limit, the above integral yields the spectrum [26]

$$\frac{dN}{dE} = \frac{24.886}{4096\pi^6}\alpha^{-1}\left(\frac{t_e}{t_0}\right)^n m_{pl}^6 E^{-3}. \tag{23}$$

(where α has been defined below Eq.(7).) The spectrum declines as E^{-3} like the standard spectrum.

Now, consider black holes evaporating at a time $t_{evap} \geq t_{dec}$. We make the approximation that all the energy gets released instantly, but take the spectrum into account. The black hole mass fraction just before evaporation is given by

$$\alpha_{evap} = \alpha_i \frac{M(t_c)}{M_i}\frac{a(t_{evap})}{a(t_i)}, \tag{24}$$

while the number density (which scales as a^{-3}) is

$$n_{evap}(M_i) = n_{evap}(M_c)\frac{a(t_c)^3}{a(t_i)^3}, \tag{25}$$

where

$$n_{evap}(M_c) = \alpha_{evap}\frac{\rho_{rad}(t_{evap})}{M(t_c)} \tag{26}$$

leading to

$$n_{PBH}(t_{evap}) = \alpha_{evap}\frac{\rho_{rad}(t_{evap})}{M(t_c)}\left(1 - \frac{3}{2}f\right)^{-3/2}. \tag{27}$$

The energy density in photons of energy E emitted between t_{evap} and $t_{evap} + dt_{evap}$ is [26]

$$dU_{tev}(E) = n_{PBH}(t_{evap})E^2\frac{dN}{dE}(t_{evap})\frac{dt_{evap}}{t_{evap}}. \tag{28}$$

We require the present total energy density in Hawking photons at a certain energy scale E_0, denoted as $U_0(E_0)$.

The photon energy E emitted at time t undergoes redshift due to expansion of the Universe. So the photons emitted at time t_{evap} with energy E is related with presently observed energy as

$$E(E_0) = E_0\frac{a(t_0)}{a(t_{evap})}. \tag{29}$$

Since

$$\rho_{rad}(t_{evap}) = \rho_{rad}(t_0)\frac{a(t_0)^4}{a(t_{evap})^4}, \tag{30}$$

one gets

$$U_0(E_0) = \int_{t_{dec}}^{t_0} dU_0(E_0) = \int_{t_{dec}}^{t_0}\frac{a(t_{evap})^4}{a(t_0)^4} dU_{tev}[E(E_0)]. \tag{31}$$

Substituting the relevant expressions and using Eq.(10), we get

$$U_0(E_0) = m_{pl}\left(\frac{t_0}{t_e}\right)^{(2-n)/3}(t_e)^{\frac{1}{2}+n}\left(\frac{t_0}{t_e}\right)^{\frac{n}{2}}(3\alpha)^{-1/2}\rho_{rad}(t_0)E_0^2 t_0^{(2-n)/3}(2n+1)^{\frac{1}{2}}$$
$$\int \alpha_i\left(1 - \frac{3}{2}f\right)^{-3}(t_{evap})^{-(\frac{1}{2}+n)}(t_{evap})^{-(2-n)/3}\frac{dN}{dE(E_0)}(t_{evap})\frac{dt_{evap}}{t_{evap}}. \tag{32}$$

The number spectrum of a black hole of initial temperature T_{BH} peaks at an energy $E = bT_{BH}$ with $b \approx 5$ [33] in the standard treatment. In the BD case the b-value remains unchanged since in its determination from the power spectrum of the emitted particles, G which in this case is $\sim \phi^{-1}$, gets cancelled. Therefore, unless α_i is sharply peaked at particular initial epochs, the main contribution to the integral in Eq.(32) is obtained when $E(E_0) = bT_{BH}(t_{evap})$, i.e., from PBHs evaporating at $t_{evap} = t_{main}$, where

$$t_{main} \approx \left(\frac{E_0}{bT_{BH}(t_0)} \right)^{3/1-3n} t_0. \tag{33}$$

The contribution from PBHs evaporating earlier will come from the high frequency end of their spectrum, while PBHs evaporating at later times will contribute radiation that originated in the low frequency end. Using $x_i = b \approx 5$, the number spectrum equation (22) becomes

$$\frac{dN}{dE(E_0)}(t_{evap}) = \frac{14.3}{4096\pi^6} \frac{m_{pl}^6}{\alpha} \left(\frac{t_e}{t_0} \right)^n E_0^{-3} \left(\frac{a(t_{evap})}{a(t_0)} \right)^3. \tag{34}$$

Using this equation in equation(32), we estimate the total energy density at energy E_0 as

$$U_0(E_0) = \frac{14.3}{4096\pi^6} \frac{m_{pl}^6}{\alpha} \left(\frac{t_e}{t_0} \right)^n m_{pl} \left(\frac{t_0}{t_e} \right)^{(4+n)/6} (t_e)^{\frac{1}{2}+n} (3\alpha)^{-1/2} \rho_{rad}(t_0) E_0^{-1} t_0^{(2-n)/3} (2n+1)^{\frac{1}{2}}$$
$$\int_{t_{dec}}^{t_{main}} \alpha_i (1 - \frac{3}{2}f)^{-3} (t_{evap})^{-(1+10n)/6} \, dt_{evap}, \tag{35}$$

which gives

$$U_0(E_0) \propto E_0^{(3-16n)/(2-6n)}. \tag{36}$$

In the standard spectrum $U_0(E_0)$ varies as $E_0^{3/2}$.

Radiation at lower frequencies will originate from the low frequency ends of the instantaneous spectra, with the dominant contribution coming from PBHs evaporating around t_{dec}. Its intensity can generically be neglected as compared to the main frequency range [26]. For energies $E_0 > bT_{BH}(t_0)$, the dominant part comes from the high frequency tail of PBHs evaporating today. The number spectrum equation in the high energy limit is used to obtain

$$U_0(E_0) = 6.16 \times 10^{54} E_0^{-1} \alpha_i \left(1 - \frac{3}{2}f \right)^{-3} \quad keV \, cm^{-3} s^{-2} keV^{-1}. \tag{37}$$

The spectral surface brightness $I(E_0)$, an observational quantity, is related to the integrated energy density $U_0(E_0)$ by [27]

$$I(E_0) = \frac{c}{4\pi} \frac{U_0(E_0)}{E_0}. \tag{38}$$

The overall peak in the present spectrum is at $E_{peak} = bT_{BH}(t_0)$. So equation(38) gives

$$I(E_{peak}) = 2.08 \times 10^{20} \left(1 - \frac{3}{2}f \right)^{-1} \alpha_i \quad keV \, cm^{-2} s^{-1} sr^{-1} keV^{-1}. \tag{39}$$

Considering a value of $E_{peak} \approx 100$ Mev [34], which is not too far from the present observational range [35] of γ-rays, we have

$$I_{obs} = 1.11 \times 10^{-5} \quad keV \, cm^{-2} s^{-1} sr^{-1} keV^{-1}. \tag{40}$$

The constraint $I(E_{peak}) < I_{obs}$ then results in an upper limit on the initial mass fraction as

$$\alpha_i < 5.34 \times 10^{-26} \times \left(1 - \frac{3}{2}f \right). \tag{41}$$

For comparison, the corresponding constraint in the standard case is obtained from the gamma-ray background at $E_{peak} = 100 MeV$ and reads $\alpha_i < 10^{-27}$ [23, 34, 41]. It may be noted here that the recent work of Carr et al. [25] confirms earlier results [36] that the spectrum of secondary photons is peaked at $E \approx 68 MeV$, independent of the PBH temperature. Our constraint on the PBH mass fraction originates from considering the peak value $E \approx 100 MeV$ for primary photons. As shown by Carr et al. [25], secondary photon emission may dominate when the PBH mass falls below the QCD scale, in which case the constraints may be altered at most by an order of magnitude. Similar considerations would apply to the Brans-Dicke case, but the exact magnitude of the constraint would require numerical analysis to be evaluated.

V. DISTORTION OF THE COSMIC MICROWAVE BACKGROUND SPECTRUM

Hawking radiation emitted at redshifts $z \leq 2 \times 10^6$ or $t \geq 4 \times 10^{-10} t_0$ can not be fully thermalised and will disturb the Cosmic Microwave Background spectrum. The corresponding modification of the Planck spectrum is described by the chemical potential μ [37], which is related with injected energy density as

$$\frac{\rho_{evap}}{\rho_{rad}(t)} = 0.71\mu, \tag{42}$$

where ρ_{evap} is the energy density injected by evaporating PBHs. Observational results [38] suggest an upper limit on μ given by

$$\mu < 9 \times 10^{-5}. \tag{43}$$

Assuming that about a half of the energy is emitted in the form of particles capable to disturb the CMB spectrum, one can write

$$\frac{1}{2}\alpha_{evap} = 0.71\mu. \tag{44}$$

Using equation (43), we get

$$\alpha_{evap} < 1.28 \times 10^{-4}, \tag{45}$$

which leads to the initial PBH mass fraction constraint as

$$\alpha_i < 1.28 \times 10^{-21} \times \left(1 - \frac{3}{2}f\right)^{3/2}. \tag{46}$$

In the standard cosmology the corresponding constraint is [39]

$$\alpha_i < 10^{-21}. \tag{47}$$

In a recent work it was shown by Tashiro and Sugiyama [40], that secondary photon emissions leading to non-zero chemical potential for photons could impact the CMB spectrum. However, it follows from the analysis of Carr et al. [25], that the constraint on the PBH mass fraction stays around the value 10^{-21}.

VI. NUCLEOSYNTHESIS CONSTRAINTS

Standard big-bang primordial nucleosynthesis is one of the most well-understood processes in the early Universe. Therefore, this era is an important benchmark to look for effects due to the interactions of particles emitted by PBHs [41]. Several detailed investigations of PBHs in standard cosmology have computed the predicted changes in the density of light elements [42, 43]. Existing observational limits on the light element abundances have then been used to put constraints on the size of such modifications,

which in turn lead to constraints on the numbers of PBHs that could evaporate both during and after nucleosynthesis. In the standard scenario, the PBHs which evaporate during nucleosynthesis with t_{evap} between 1s to 400s, have initial masses in between 10^9g and 10^{10}g. This remains nearly the same in Brans-Dicke theory where the initial mass varies between $3.1 \times 10^9 \times (1 - \frac{3}{2}f)$g and $2.28 \times 10^{10} \times (1 - \frac{3}{2}f)$g. Following again the analysis of Clancy, Guedens and Liddle [26] (performed in the context of braneworld black holes) here we examine two nucleosynthesis constraints in the context of the Brans-Dicke theory, namely the constraint on the increase in production of helium-4 due to the injection of PBH hardons [43, 44] and the constraint on the destruction of primordial deuterium by PBH photons [43, 45]. Note here that the nucleosynthesis constraints are the ones that could be most affected by taking into account quark and gluon emission by PBHs. Inter-conversion between protons and neutrons due to emitted mesons and anti-nucleons increases the n/p freeze-out ratio as well as the final He-4 abundance, as shown by Carr et al. [25] in their recent work. Our main aim here is to get a reasonable estimate of how such constraints are modified in BD theory.

A. The Helium abundance constraint

The total number density of emitted particles from the complete evaporation of PBHs of some initial mass may be expressed as

$$N_{em} = \frac{\rho_{PBH}}{< E_{em} >}, \tag{48}$$

where $< E_{em} >$ is the average energy of the emitted particles. The ratio of the energy density in PBHs at evaporation to the background radiation energy density is therefore

$$\frac{\rho_{PBH}}{\rho_{rad}} = \alpha_{evap} = \frac{< E_{em} > N_{em}}{< E_{rad} > N_{rad}}, \tag{49}$$

where $< E_{rad} >$ and N_{rad} are the average energy and number density of the particles comprising the background cosmological radiation fluid. The ratio of the average energies can be approximated by the ratio of the PBH temperature at the onset of evaporation to the background temperature at evaporation, i.e.,

$$\frac{< E_{em} >}{< E_{rad} >} = \frac{T_{BH}}{T_{evap}}. \tag{50}$$

Using equation (2)–(8) and applying the standard cosmological temperature-time relation [46]

$$t = 0.301 g_*^{-1} \frac{m_{pl}}{T^2} \tag{51}$$

(where g_* is a constant having value 10.78), one gets

$$\frac{< E_{rad} >}{< E_{em} >} = \frac{T_{evap}}{T_{BH}} = 7.28 \times \left(1 - \frac{3}{2}f\right)\left(\frac{M(t_c)}{m_{pl}}\right)^{-1/2}. \tag{52}$$

The total emitted number density at time t_{evap} during nucleosynthesis thus becomes

$$N_{em} = 7.28\left(1 - \frac{3}{2}f\right)\alpha_{evap}\left(\frac{M(t_c)}{m_{pl}}\right)^{-1/2} N_{rad}. \tag{53}$$

Using observational estimations, Clancy *et. al.*[26] found that

$$N_{em} < \frac{2.8}{100F} n_b, \tag{54}$$

where F is the fraction of the total particles emitted by PBH having value $F \leq 0.2$. Comparing above two equations, one can get

$$\alpha_{evap} < \frac{0.38}{F} \times 10^{-2} \left(1 - \frac{3}{2}f\right)^{-1} \left(\frac{M(t_c)}{m_{pl}}\right)^{1/2} \eta_{evap},$$

(55)

where $\eta_{evap} = \frac{n_b}{N_{rad}}$ is the baryon to photon ratio at evaporation. Assuming that η is fixed from evaporation up to present times, i.e., $\eta_{evap} = \eta_0$ and using the relation $\eta_0 \approx 2.8 \times 10^{-8} \Omega_b h^2$ with $\Omega_b h^2 \approx 0.02$, Eq.(55) becomes

$$\alpha_{evap} < 1.064 \times 10^{-11} \times \left(1 - \frac{3}{2}f\right)^{-1} \left(\frac{M(t_c)}{m_{pl}}\right)^{1/2}.$$

(56)

The bounds imposed at nucleosynthesis can be converted into bounds on the initial PBH mass fraction for PBHs that evaporate at $t_{evap} = 400$s:

$$\alpha_i < 8 \times 10^{-20} \times \left(\frac{M_i}{10^9 g}\right)^{1/2}.$$

(57)

One may compare the constraint with that of standard cosmology [43, 44]

$$\alpha_i < 3 \times 10^{-18} \left(\frac{M_i}{10^9 g}\right)^{1/2}.$$

(58)

Equation(57) further leads to

$$\alpha_i < 3.82 \times 10^{-19} \times \left(1 - \frac{3}{2}f\right)^{1/2}.$$

(59)

It turns out from the analysis by Carr et al. [25] that by considering the effects of quark and gluon emission by the PBHs, an earlier constraint [43] on the PBH mass fraction is actually weakened in the relevant mass range that we are also considering for our analysis in the Brans-Dicke case.

B. Deuterium photodisintegration constraint

The high-energy particles emitted by evaporating PBHs both during and after nucleosynthesis can be sufficiently energetic to disrupt primordial nuclei. One important reaction of this type is photodisintegration which entails the destruction of primordial nuclei by high-energy PBH photons. Of all the primordial nuclei, deuterium is the most susceptible to photon-disintegration. If ΔM is the PBH mass evaporated between times t_1 and t_2 during which deuterons are destroyed and M_b is the baryonic mass, then [45]

$$\frac{\Delta M}{M_b} \leq \frac{\epsilon}{f_\gamma \beta} \frac{E_*}{m_p},$$

(60)

where f_γ is the fraction of mass that decays into photons, ϵ is the depletion factor, E_* and β are constants. t_1 and t_2 are taken to be the end of nucleosynthesis and the onset of recombination respectively. We estimate ΔM by the PBH mass evaporated shortly after nucleosynthesis. It is usually justified to take [45]

$$\frac{\Delta M}{M_b} = \left[\frac{\rho_{PBH}}{\rho_b}\right]_{t_{evap}}$$

(61)

with t_{evap} some time after nucleosynthesis, either when a narrow mass range of PBHs evaporates, or straight after nucleosynthesis for an extended mass spectrum. Equation (60) then leads to

$$\alpha_{evap} \leq \left[\frac{\rho_b}{\rho_{rad}}\right]_{t_{evap}} \frac{\epsilon}{f_\gamma \beta} \frac{E_*}{m_p}.$$

(62)

Since $\rho_b \propto a^{-3}$ and $\rho_{rad} \propto a^{-4}$, it follows that

$$\left[\frac{\rho_b}{\rho_{rad}}\right]_{t_{evap}} = \frac{a(t_{evap})}{a(t_e)}\left[\frac{\rho_b}{\rho_{rad}}\right]_{t_e} = 2\frac{a(t)}{a(t_e)}\Omega_b(t_e) \tag{63}$$

as $\rho_{rad} = \rho_{tot}/2 \approx \rho_c/2$ at the time of matter radiation equality $t_e = 10^{11}$s. The baryon density parameter at equality is related to the present one, as the matter density parameter at equality is given by $\Omega_m(t_e) \approx 1/2$.

$$\Omega(t_e) = \frac{\Omega_m(t_e)}{\Omega_m(t_0)}\Omega_b(t_0) \approx \frac{1}{2}\frac{\Omega_b(t_0)}{\Omega_m(t_0)}. \tag{64}$$

Using equation (64) in equation (62), one gets

$$\alpha_{evap} \leq \frac{\epsilon}{f_\gamma \beta}\frac{E_*}{m_p}\left(\frac{t_{evap}}{t_e}\right)^{1/2}\frac{\Omega_b(t_0)}{\Omega_m(t_0)}. \tag{65}$$

Using numerical values of constants [26] $f_\gamma = 0.1$, $E_* = 10^{-1}$Gev, $\beta = 1$, $\epsilon \sim 1$ and taking the ratio of the present baryonic density to the total matter density as 0.1, we get

$$\alpha_{evap} \leq 1.066 \times 10^{-28}\left(\frac{t_{evap}}{t_{pl}}\right)^{1/2}. \tag{66}$$

For $t_{evap} = 400$ sec, the constraint becomes

$$\alpha_{evap} \leq 6.74 \times 10^{-6}, \tag{67}$$

which gives

$$\alpha_i \leq 5.1 \times 10^{-21} \times \left(1 - \frac{3}{2}f\right)^{3/2}. \tag{68}$$

In standard case this constraint is [43, 45]

$$\alpha_i \leq 10^{-21}. \tag{69}$$

The recent analysis by Carr et al. [25] reveals that the standard deuterium constraint [43] is tightened by two orders of magnitude, since hadrodissociation of helium due to injected nuclei produces more deuterium. The same is expected to be true for the Brans-Dicke scenario, but, as in the earlier cases, only detailed numerical simulations could reveal the actual quantitative changes on the constraints.

VII.   SUMMARY AND CONCLUSIONS

In this paper we have analyzed several astrophysical constraints on the initial mass fraction of primordial black holes that could grow due to accretion of radiation in the early Universe in Brans-Dicke cosmology. It is well-known that PBHs loose mass due to Hawking evaporation with their lifetime depending upon their initial masses. However, the accretion of radiation could be an efficient process in the BD formalism, depending upon the accretion efficiency, leading to the growth of mass and thereby enhancing their lifetime [18, 21]. Such PBHs, once formed, will influence the later cosmological epochs through their total density, and also through the products of their evaporation. Since the standard cosmological scenario is based on a sound observational footing, at least from the era of nucleosynthesis onwards, any modification due to surviving PBHs at various eras is tightly constrained by various observational results, such as the photon spectrum, CMB radiation, light element abundances, etc. Such constraints, in turn, can be translated into constraints on the initial PBH mass fraction. This translation is contingent on the particular cosmological evolution, and is thus sensitive to the theory of gravity considered, as has been

TABLE I: The variation of the upper bound of the initial PBH mass fraction(α_i) with the accretion efficiency f for different cases is shown in the table

Cause of the Constraint	$f = 0$	$f = 0.25$	$f = 0.45$	$f = 0.65$
Present Density	3.43×10^{-18}	1.69×10^{-18}	0.62×10^{-18}	0.01×10^{-18}
Photon Spectrum	5.34×10^{-26}	3.34×10^{-26}	1.73×10^{-26}	1.33×10^{-27}
Distortion of CMB	1.28×10^{-21}	0.63×10^{-21}	0.23×10^{-21}	0.05×10^{-22}
Helium abundance	3.82×10^{-19}	3.01×10^{-19}	2.18×10^{-19}	0.60×10^{-19}
Deuterium abundance	5.10×10^{-21}	2.52×10^{-21}	0.94×10^{-21}	0.02×10^{-21}

shown earlier in the context of BD gravity without accretion [20], and also in the context of braneworld gravity [26]. Moreover, inclusion of the effect of accretion also impacts upon the constraints, as seen earlier in the context of braneworld gravity [26], and also as shown by us in the present work.

The summary of our results are presented in Table 1. Here we enlist the upper bounds on the initial mass fraction of PBHs that are allowed by taking into considerations various observational features as listed. These constraints obviously depend upon the accretion efficiency, as seen from the displayed numbers. The initial constraints are usually the most severe for those black holes whose lifetimes are comparable with the cosmic time of the epoch at which the observational constraint is imposed, and our displayed results correspond to such cases. We consider three different values of the accretion efficiency, with $f \approx 2/3$ corresponding to the maximum allowed in BD theory. The $f = 0$ case corresponds to the BD formalism without accretion. It is seen that in all cases the inclusion of accretion strengthens the constraint on the upper bound of the initial PBH mass fraction.

Since the present observable Universe is flat, the mass density of presently surviving PBHs should not exceed that of dark matter which is about 0.3 times the critical density. This imposes a constraint on initial PBH mass fraction which is of order 10^{-18} for lower accretion efficiencies and grows by two orders for higher accretion efficiencies. The standard cosmology constraint is of order 10^{-18}. PBHs which evaporate between photon decoupling and the present age leave behind a spectrum that peaks at a temperature of the order of the black hole temperature at the onset of evaporation of PBHs with $t_{evap} \approx t_0$. In this case the bound on the initial PBH mass fraction is comparable with standard cosmology for higher accretion efficiencies and grows weaker by one order for lower accretion efficiencies. If evaporation products are released around the Sunyayev-Zel'dovich time $t_{SZ} \approx 4 \times 10^{-10} t_0$, they will fail to fully thermalize the background radiation. This time, however, is sufficiently early for the excess energy to distort the background blackbody spectrum. Limits on the allowed distortion of the CMB spectrum then imply limits on PBH mass fraction. For lower accretion efficiencies this constraint is of the same order as in standard cosmology, but increases more than three orders for maximally efficient accretion. If there were a population of PBHs evaporating during or after the era of nucleosynthesis, this could have led to significant change in the final light element abundances. Considering the Helium abundance as an example, we have found that the constraint on the initial PBH mass fraction once again varies as it started from a value of nearly one order less compared to standard cosmology and grows closer to it as accretion efficiency increases. Considering photon-disintegration and change in deuterium abundance, we find that the initial constraint is nearly of the same order as the standard value for low accretion efficiencies, increasing by nearly two orders for higher values. Comparing the constraints due to the different observational features considered in this work, we note that the photon spectrum imposes the most stringent limits on the initial PBH mass fraction.

We conclude by emphasizing that Brans-Dicke cosmology which provides a viable alternative to the standard scenario, imposes upper bounds on the allowed initial mass fraction of primordial black holes, that are modified compared to standard cosmology. Depending upon the particular observed physical process used to impose the constraints, these upper bounds in BD gravity could either be strengthened or weakened compared to the case of standard gravity [20]. It needs to be mentioned here that some of the constraints of standard gravity could themselves be modified by considering effects of quark and gluon emission and the resultant emission of secondary photons by PBHs, as discussed in the recent work by Carr et al. [25]. However, as shown in the present paper the inclusion of the effect of accretion tightens

the constraints in all cases since PBHs in BD gravity could start with a lower value of initial mass, and subsequently grow in size sufficiently [21] to impact the observational features in future eras. Finally, it remains to be seen how additional effects such as considering PBHs in further general scalar-tensor models of gravity such as in [18], or taking into account the effects of backreaction of the PBHs on cosmological evolution, could modify the observational constraints on the initial mass spectrum.

Acknowledgements

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