Asymptotic properties of restricted naming games

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HIGHLIGHTS

- Naming games are studied with finite sizes of the agent vocabularies.
- Naming games are studied with limited number of distinct names.
- Different dynamical rules lead to different new power law exponents.

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Naming games
Self-organized systems
Scaling
Critical exponents
Structures and organization in complex systems
Critical phenomena

ABSTRACT

Asymptotic properties of the symmetric and asymmetric naming games have been studied under some restrictions in a community of agents. In one version, the vocabulary sizes of the agents are restricted to finite capacities. In this case, compared to the original naming games, the dynamics takes much longer time for achieving the consensus. In the second version, the symmetric game starts with a limited number of distinct names distributed among the agents. Three different quantities are measured for a quantitative comparison, namely, the maximum value of the total number of names in the community, the time at which the community attains the maximal number of names, and the global convergence time. Using an extensive numerical study, the entire set of three power law exponents characterizing these quantities are estimated for both the versions which are observed to be distinctly different from their counter parts of the original naming games.

1. Introduction

The aim of the model of naming game is to study the evolution of consensus opinion in the context of naming a single object in a large community of agents [1,2]. Different agents refer to the object using different names when the object is introduced initially. Agents interact among themselves and share the names that have been already introduced according to a set of specific rules. At the early stage, the number of distinct names for the object increases as the agents introduce new names for the object. However as the game progresses, a consensus name gradually emerges and distinct names disappear. The dynamical evolution of the game terminates when all agents agree upon a single name through mutual interactions and following the rules of the game.

At an arbitrary intermediate stage, an agent has a number of names of the same object in his vocabulary suggested by different groups of agents. An agent, under the sharing dynamics, not only learns new names for the object but also shares names from his own vocabulary with other agents. In the models studied for the dynamics of naming games in the literature, the sizes of the vocabularies of the agents have been assumed to be infinite [3–10]. Real world data in this problem

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have been analyzed in [11]. It has also been shown that faster convergence can be attained in presence of overhearers [12]. Categorization of different names and color naming have been studied as well [13–15].

However, in reality, an individual agent has only a finite amount of memory. Therefore, it would be quite appropriate to study the effect of finiteness of the vocabulary sizes in the dynamics of naming games. In this work, the vocabulary size of every agent is assumed to be finite and has been restricted to a certain fixed cut-off value, which has been suitably tuned.

The dynamics of naming game is defined in terms of a sequence of bipartite interactions in a community of N agents. A 'pseudo' time t is defined for the convenience of following the dynamics, which is equal to the number of bipartite interactions. In each interaction, a pair of distinct agents is randomly selected and are allowed to interact. At any intermediate time, the vocabulary of each agent i is likely to have some entries and is denoted by the set $\{\ell_i(t)\}$ of size $\ell_i(t)$. Commonly, the naming dynamics is described in terms of a few quantities. For example, the total number of names $W(t, N) = \Sigma_{i=1}^{N} \ell_i(t)$ with all agents at time t is a well known quantity to look at. Typically, this number initially grows with time, reaches a maximum, which then gradually decreases and finally converges to N. The maximum value of W after the configuration averaging grows with N as a power law like,

$$\langle W_m(N) \rangle \sim N^{\gamma}. \tag{1}$$

At the same time, it is also customary to define two different time scales associated with the evolution dynamics. One is the time t_m when W reaches its maximum value. Again, a configuration averaged value of this quantity grows like

$$\langle t_m(N) \rangle \sim N^{\alpha}. \tag{2}$$

Secondly, one defines the convergence time t_f when every agent has only one name in his vocabulary, the same name for all agents and therefore the entire community has only N names. The averaged value of such a time scale is also assumed to vary like

$$\langle t_f(N) \rangle \sim N^{\beta}. \tag{3}$$

The three exponents in the power, namely α, β and γ, characterize the naming game. In the version of the game stated above, an agent can interact with any other agent. Therefore it is a mean field model. Yet, we have seen in the literature that the values of the above set of exponents do depend on few detailed features of the dynamical rules, as mentioned below.

In Section 2, we have studied both the symmetric and asymmetric naming games with restricted size of the agents' vocabulary and observe that the set of exponents cross over to a new set of values not studied earlier in the literature. Further in Section 3, we have studied the effect of limiting the initial number of names that are assigned to the agents. In Section 4 we summarize.

2. Symmetric and asymmetric naming games with restricted vocabulary

The naming game is defined in terms of a community of N agents and a new object to name [1]. An agent, either invents a new name for the object, or he learns a name from another agent by the bipartite sharing dynamics. Finally, all the agents come to an agreement and refer the object by a single name. This spontaneous evolution of a consensus name is the objective of the naming game. During the time evolution, pairs of randomly selected distinct agents execute a sequence of interactions to share their stock of the names of the object.

In the literature, two models of naming game have been studied. In the original 'asymmetric' naming game, one of the two agents selected for sharing, say the 'i'th agent, is called the 'speaker', where as the 'j'th agent is termed as the 'hearer' [1]. The speaker first randomly selects a name from his vocabulary and checks if the hearer also has the same name. If the hearer has this name it is called a successful sharing and then the vocabulary sizes of both the agents are reduced to unity, both having only the selected name. On the other hand, in case of a failure, the selected name of the speaker is added to the vocabulary of the hearer.

In comparison, in the 'symmetric' naming game [10], there is no distinction between the speaker and the hearer. Here, for a successful move, the entire subset of names that are common in the vocabularies of the agents i and j are retained, and the remaining un-common names are deleted from the vocabularies of both the agents. On the other hand, in case of a failure there is no common name and both the agents get the combined list of both agents' vocabularies. There are studies of similar symmetric exchange of informations in the literature [16–19].

We have studied the effect of restrictions on both the symmetric and asymmetric naming games. We describe the game and present the plots of the data for the symmetric naming game only. The plots of the asymmetric naming game exhibit very similar type of behavior. However, the power law exponents of both the games are found to be distinctly different and we have enlisted them in Table 1.

2.1. The model

In the symmetric game, we first abolish the step for the invention of names. Instead, we assign every agent a distinct name at the initial stage. Therefore, $\ell_i(t) = 1$ for all i at time $t = 0$ and the dynamics starts with N such distinct names. Further, we apply a restriction to the vocabulary size of every agent. The vocabulary size is assigned a maximal cut-off value s, same

Table 1
Comparison of the values of different exponents obtained in the restricted asymmetric and symmetric naming games with similar exponents of the original naming games.

Naming game	α	β	γ
Asymmetric game [1]	1.5	1.5	1.5
Symmetric game [10]	1.12	1.14	1.539
Rest. vocabulary (Symm)	1.34(1)	1.60(1)	1.00(1)
Rest. vocabulary (Asymm)	1.87(1)	2.18(1)	1.50(1)
Rest. names	1.09(1)	1.10(1)	0.99(1)

for all agents at all times, and no agent can accommodate any additional name. We follow the rules of symmetric naming game [10] to describe the dynamics. At every time step two distinct agents are randomly selected, the first is called the *i*th agent and the second is called the *j*th agent. These two agents are allowed to interact between themselves. The interaction can be of two types according to which both the agents update their vocabularies.

A. *Failure*: In this case, none of the names in the vocabulary of the *i*th agent is common to the vocabulary of the *j*th agent. Therefore, the agents share their vocabularies entirely. Each agent gets the combined list of names of both the agents. In mathematical form:

If $\{\ell_i(t-1)\} \cap \{\ell_j(t-1)\} = \varnothing$, then

$\{\ell_i(t)\} = \{\ell_j(t)\} = \{\ell_i(t-1)\} \cup \{\ell_j(t-1)\}$, where $\varnothing$ is the empty set.

However, if $\ell_i(t-1) + \ell_j(t-1) > s$, then a list of names of length $\ell_i(t-1) + \ell_j(t-1)$ is made in which the first $\ell_i(t-1)$ names are from the vocabulary of the *i*th agent and the remaining $\ell_j(t-1)$ names are from the vocabulary of the *j*th agent. Once this list is prepared, only the first *s* names are retained in the vocabularies of both the agents.

B. *Success:* In this case, at least one of the names in the vocabulary of the agent *i* is common to the vocabulary of the agent *j*. Then, after sharing both the agents retain only their common names.

If $\{\ell_i(t-1)\} \cap \{\ell_j(t-1)\} \neq \varnothing$, then

$\{\ell_i(t)\} = \{\ell_j(t)\} = \{\ell_i(t-1)\} \cap \{\ell_j(t-1)\}$.

Here, the finite size *s* of the vocabularies does not affect this sharing step. There are many studies that speaks a volume about finite memory [16–19]. In the following articles, the finiteness of both the short term and long term memory has been presented. In the study of [17,18] it has been observed from a set up experiment that, upto a finite number ($\approx$8) of words can be remembered exactly in the short term memory. In [19], it has been shown that there is a limit to learn and the per day limit is about 60 words. Its an ideal scenario that agents are naming a single object throughout the gaming dynamics. However, practically, agents might be involved in many such games to name many objects. Hence the limit of learning, per day per object, becomes finite and really small. These facts justifies the assumption of a finite memory in the context of naming games.

2.2. *Results*

Initially, each agent has only one name in his vocabulary which is distinct from the names of all other agents. As time passes, gradually every agent learns names from the other agents and his vocabulary gets larger. Soon, a rapid growth of the total number of names $W(t, s, N)$ takes place. On the other hand, the total number of distinct names $D(t, s, N)$ starts from its initial maximum value *N* and gradually decreases. At a certain intermediate time it decays fastest. In a specific run of the naming game, the $W(t, s, N)$ reaches its maximum value $W_m(s, N)$ at the maximal time $t_m(s, N)$. Finally, in the long time limit the community reaches a fixed state where all agents have the same set of names, with one or few elements. The time at which the community reaches this state is referred as the convergence time $t_f(s, N)$. Using a large number of such independent runs we have calculated the averaged values of $\langle W(t, s, N)\rangle$, $\langle D(t, s, N)\rangle$, $\langle W_m(s, N)\rangle$, $\langle t_m(s, N)\rangle$ and $\langle t_f(s, N)\rangle$ for different community sizes *N*.

In Fig. 1(a), the time variation of the averaged total number of names $\langle W(t, s, N)\rangle$ present in the community has been shown. For smaller vocabulary sizes $s = 2, 5, 10$ etc., this number quickly reaches its maximum value *sN*, because each individual agent's vocabulary gets filled up. This maximum value is then maintained for some time, accordingly the curve has a plateau over a relatively longer period. Smaller the value of *s* longer is the length of the plateau. At a certain initial time t_{in}, $W(t, s, N)$ attains the value *sN*, remains equal, or very close to it for a long time and then at a latter time t_{out}, $W(t, s, N)$ decreases from *sN*. Both t_{in} and t_{out} fluctuate over a wide range. We have therefore defined the maximal time t_m for attaining the maximum number of names in the community as mean of these two reference times, i.e., $t_m(s, N) = (t_{in} + t_{out})/2$.

However, for the same *N*, but for relatively larger values of the cut-off sizes like $s = 80$ or 160, the plateau gradually shrinks and the number of names with an agent hardly reaches its cut-off value *s*. Instead, a single peak of value $\langle W_m(s, N)\rangle$ starts showing up at $\langle t_m(s, N)\rangle$ as *s* increases even further. The unrestricted case of $s \to \infty$ is the original symmetric naming game (Fig. 2). In this limit, the behavior is not much different from the same result in the symmetric naming game [10]. From the averaged slopes of these lines, our estimate for the exponent γ is 1.51. However, on systematic extrapolation of slopes between successive points yields an even larger value of 1.54 which is very close to $\gamma = 1.539$ of the symmetric naming game [10]. On the other hand, when *s* is very small the exponent is found to be $\sim$1.00(1). The corresponding γ exponents of the asymmetric naming game are 1.50 and 1.00 respectively.

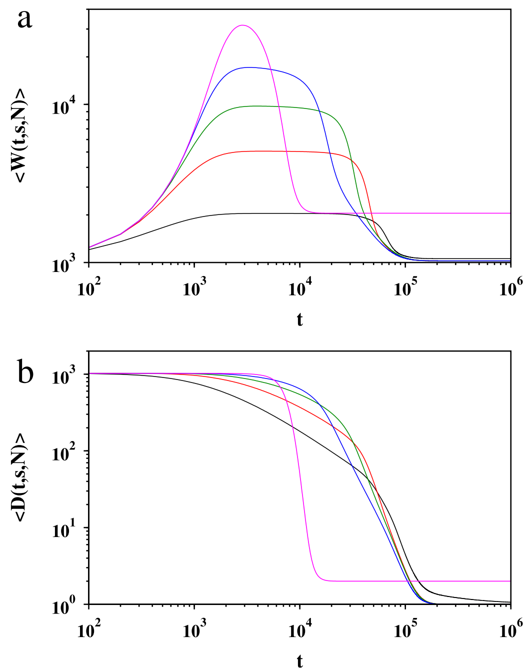

Fig. 1. (a) The total number of names $\langle W(t, s, N)\rangle$ averaged over many independent runs, for a community of fixed population size $N = 1024$ with the restricted vocabulary sizes s for the agents, has been plotted against time t. Colors are: $s = 2$ (black), 5 (red), 10 (green), 20 (blue) and ∞(magenta). After a slow start, the $\langle W(t, s, N)\rangle$ grows sharply with time, reaches a plateau, and then decreases gradually and finally converges to a value N, when every agent has the one and the same name. For the unrestricted case, the convergence value is $2N$. (b) The total number of distinct names $\langle D(t, s, N)\rangle$ for the same simulation which takes the value of unity (two for $s = \infty$) at the convergence point. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

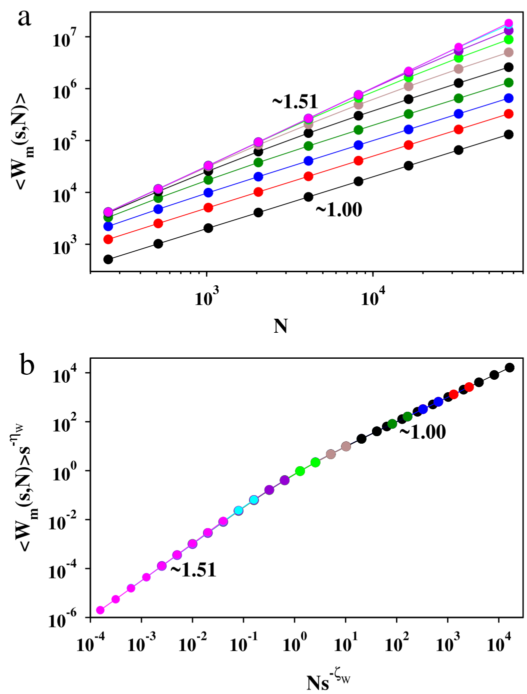

Fig. 2. (a) The configuration averaged value of the maximum number $\langle W_m(s, N)\rangle$ of names has been plotted with the community size N for different values of the vocabulary sizes: $s = 2$ (black), 5 (red), 10 (blue), 20 (green) etc. and ∞ (magenta) (from bottom to top). Clearly, for an arbitrary s, two regimes are distinctly visible. For small N and for large N the slopes are different and there is a crossover at a certain value N_c of N. (b) A re-plot of the same data, when the axes are scaled as $\langle W_m(s, N)\rangle s^{-\eta_W}$ and $Ns^{-\zeta_W}$ when the scaling parameters η_W and ζ_W are tuned to 3.0 and 2.0 respectively for the best collapse of the data. It implies that the crossover community size grows as $N_c(s) \sim s^2$ and the values of the exponent γ are $\sim$1.51 for $N \ll N_c$ and $\sim$1.00 for $N \gg N_c$ respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

For a specific value of the community size N, there is a crossover value $s_c(N)$ for vocabulary size s such that if $s \gg s_c(N)$, the dynamical evolution of the restricted naming game would not be affected at all by the restricted size of the vocabulary. The behavior of the game remains same as the unrestricted symmetric naming game. On the other hand when $s \ll s_c(N)$, the restriction is too strong, and the restricted size of the vocabulary does affect the dynamics, and consequently the set of exponents characterizing the game. Let us assume a general functional dependence $s_c(N) = \mathcal{G}(N)$. Such a relation can also be inverted and one gets

$$N_c(s) = \mathcal{G}^{-1}(s). \tag{4}$$

An immediate interpretation of this equation is, for a given value of s, if $N \ll N_c(s)$, the behavior would be similar to the unrestricted one, where as if $N \gg N_c(s)$ the effect of restriction would be strongly felt. To verify this idea quantitatively, we have performed a scaling analysis of the data used in Fig. 2(a). After a suitable tuning of the parameters η_W and ζ_W, we see a nice collapse of the data when $\langle W_m(s, N)s^{-\eta_W}\rangle$ has been plotted against $Ns^{-\zeta_W}$ in Fig. 2(b). The best values of scaling exponents are $\eta_W = 3.0$ and $\zeta_W = 2.0$ respectively. Therefore, one writes the scaling form

$$\langle W_m(s, N)\rangle \sim s^{\eta_W} \mathcal{F}\left[N/N_c(s)\right] \tag{5}$$

where, the cross-over community size is $N_c(s) = s^{\zeta_W}$. For $N < N_c(s)$ the exponent γ is approximately 1.54 and for $N > N_c(s)$, it is 1.0. Therefore, as the community size N is gradually increased, the game switches over from one behavior to the other at the crossover community size.

It is also interesting to see how the distinct number of names in the community varies with time. Fig. 1(b) exhibits the time variation of the averaged distinct number $\langle D(t, s, N)\rangle$ of names for a community size N and for the same values of s studied in Fig. 1(a). Curves for all values of s start from $\langle D_m(s, N)\rangle = N$ at time $t = 0$ and then monotonically decrease. Larger the value of s, sharper is the fall. After a long time, the number of names converges to either only one or two names.

Here we notice that at any stage, if all agents have the same set of names in their vocabularies, it becomes a stable state. Bipartite interactions between all possible agent pairs are then always successful. In such an interaction, a pair of agents retains the same set of names. Such a state does not change with time and therefore it is the convergence state. Evolution to such a state with only two names in the vocabulary can be described in the following way. Let the agents i and j have single names 1 and 2 respectively. After an interaction between them, both the agents get names $\{1, 2\}$. Further, when the agent i interacts with a third agent k, having name 3, then both i and k get the set of three names $\{1, 2, 3\}$, which includes the subset $\{1, 2\}$. In this way, different pairs of names are formed and they remain as pairs, due to the basic bipartite mechanism of the interaction. Finally one of these pairs survives.

It has been also observed that when the cut-off in the size of the vocabulary is small, the converged state almost always has only one name. On the other hand for unrestricted vocabularies, the converged state having the same pair of names with all the agents occurs with high probability. When the cut-off size s is gradually increased, the averaged number $\langle W_f(s, N)\rangle/N$ of names per agent in the converged state has been calculated and plotted in Fig. 3(a). This average value systematically increases from 1.0 to 2.0 but the entire curve shifts to the larger regimes of s values with increasing N. Using the method of interpolation we have calculated $s_0(N)$, which is the value of s for which $\langle W_f(s, N)\rangle/N = 3/2$. Plotting on a log – log scale, it is found that $s_0(N)$ scales with N as $N^{0.64}$. Further, in Fig. 3(b), we have shown a scaling of $\langle W_f(s, N)\rangle/N$ against $[s - s_0(N)]N^{-0.5}$ which exhibits an excellent data collapse.

In Fig. 4(a), $\langle t_m(s, N)\rangle$ has been plotted against N, for the same values of s using a log – log scale. Again, it is observed, that for very large, e.g., $s = \infty$ and for very small values of s, e.g., $s = 2$ the curves fit to straight lines. Slopes of these straight lines measure the values of α and they are approximately 1.12(1) and 1.34(1) respectively in these two limiting cases. For the intermediate values of s and for small community sizes the slopes are nearly 1.12 for small values of N but they gradually increase to 1.34 as N increases. Therefore, in this case also, there is a crossover from the original symmetric naming game behavior to the behavior of restricted symmetric naming game. This observation prompted us to perform a similar scaling analysis in Fig. 4(b) with the proper scaling of the abscissa and the ordinate. After tuning the scaling exponents for the best data collapse, we have obtained the scaling exponents $\eta_m = 3.0$ and $\zeta_m = 2.53$. The crossover community size therefore increases with the cut-off in the vocabulary size as: $N_c(s) \sim s^{\zeta_m}$. For the asymmetric naming game, the corresponding α exponents are 1.54 and 1.87 respectively.

Finally, the averaged convergence time $\langle t_f(s, N)\rangle$, taken by the community to reach its consensus state where all individual agents have the same set of names in their vocabularies, also shows a power law dependence on the community size N associated with the exponent β as mentioned in Eq. (3). In Fig. 5(a), we have plotted $\langle t_f(s, N)\rangle$ against N using a log – log scale. These curves fit to straight lines very closely and slopes of these lines give the estimates for the exponent β. For example, for the smallest vocabulary size ($s = 2$), $\beta = 1.60(1)$ where as for $s = \infty$ we get back the exponent of the original symmetric naming game $\beta = 1.14(1)$. Therefore, a crossover between the two behaviors is present in this case as well. For a specific value of the cut-off parameter s, which is neither very small, nor very large, we obtain $\beta = 1.14$ for $N \ll N_c(s)$, where as for $N \gg N_c(s)$ we get $\beta = 1.60$. For the asymmetric naming game the corresponding β exponents are 2.18 and 1.48 respectively.

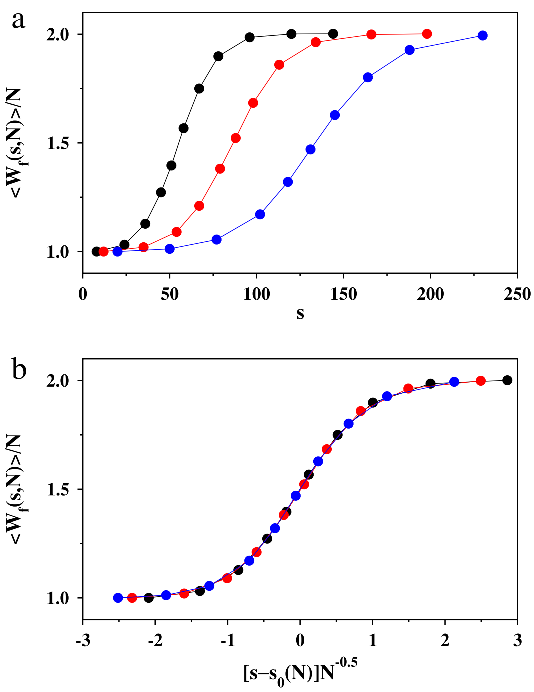

Fig. 3. (a) The configuration averaged value of the number $\langle W_f(s, N)\rangle$ of names in the converged state has been plotted against the cut-off in the vocabulary size s for three values of the community size $N = 512$ (black), 1024 (red), and 2048 (blue). (b) Data collapse for the finite-size scaling of $\langle W_f(s, N)\rangle/N$ against $[s - s_0(N)]N^{-0.5}$; $s_0(N)$ is found to grow as $N^{0.64}$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

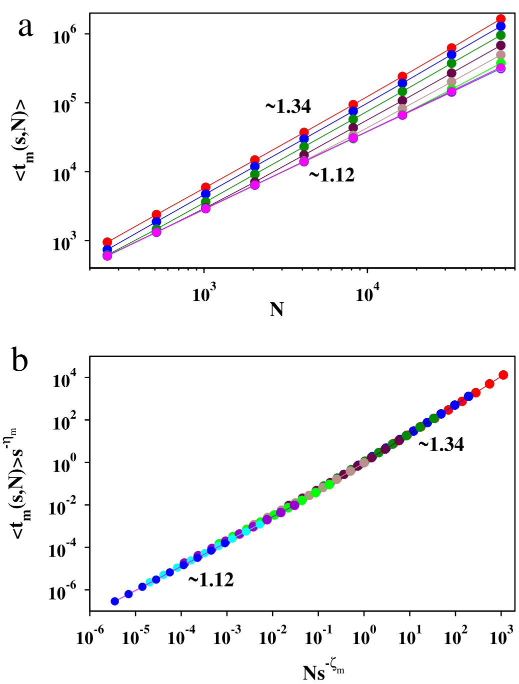

Fig. 4. (a) The averaged value of the maximal time $\langle t_m(s, N)\rangle$ has been plotted against N using the same set of vocabulary sizes and colors as used in Fig. 2. For a specific value of the vocabulary size s, the curve is a straight line with slope $\sim$1.12 for $N \ll N_c$ and $\sim$1.34 for $N \gg N_c$. (b) Again a scaling of the axes leads to a data collapse when the scaling exponents have been tuned to $\eta_m = 3.0$ and $\zeta_m = 2.53$ respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

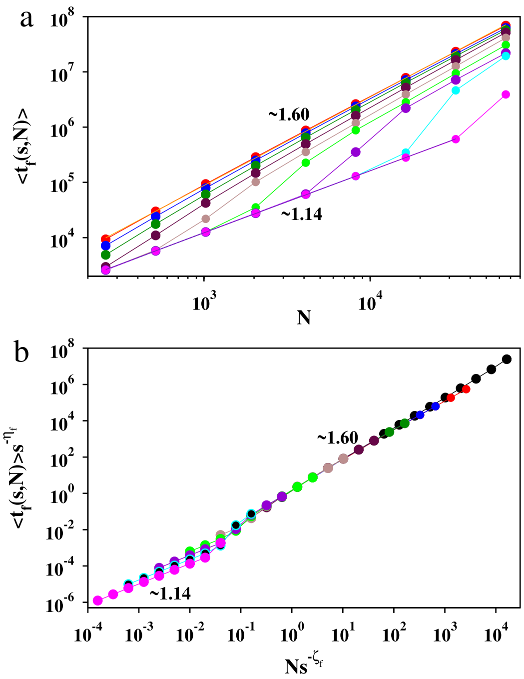

Fig. 5. (a) The averaged value of the final convergence time $\langle t_f(s,N)\rangle$ has been plotted, against N using the same set of vocabulary sizes and colors as used in Fig. 2. For a specific value of the vocabulary size s, the curve is a straight line with slope ~ 1.14 for $N \ll N_c$ and ~ 1.60 $N \gg N_c$. (b) Again a scaling of the axes leads to a data collapse when the scaling exponents have been tuned to $\eta_f = 3.0$ and $\zeta_f = 2.0$ respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

3. Symmetric naming game with limited number of distinct names: Model and results

In this section, we consider the second modification of the symmetric naming game. Initially, each agent starts with a single name in his vocabulary, but the names assigned to the agents are no longer distinct. Only $n < N$ distinct names are distributed. One distinct name is assigned to each member of a set of int$[N/n]$ agents and when all names are exhausted in this way, the remaining agents are randomly assigned one of the n names each. At any intermediate time, an agent can have in general any number of names between 1 and n. Therefore, though the size of the vocabulary is infinite in principle, an agent's vocabulary can be filled only up to n names, since no name in the vocabulary can occur more than once.

For example, for the case of $n = 2$, let the two names be 1 and 2. Therefore, an agent can be in three distinct possible states; he can have only one name, either 1 or 2, or both the names 1 and 2. Initially, all N agents are assigned randomly any of these two names with equal probability 1/2.

In this version of the game, the dynamical rules of the symmetric naming game are followed. The set of names in the vocabularies of agents evolve by mutual pairwise interactions. There is no possibility of name invention and therefore, every interaction has only the following two possibilities: failure and success. At an arbitrary time t, a distinct pair of agents i and j is first selected randomly. Interaction between the agents takes place by following the rules mentioned in Section 2.1.

If the number of agents is very large, so that $N \gg n$, then the average number of names an agent can have is also $\approx n$. This immediately implies that $\langle W_m(n,N)\rangle$ must be proportional to N. Therefore, for a fixed value of n, the growth exponent γ from Eq. (1) must be equal to unity. This is verified numerically in Fig. 6(a).

On the other hand, the case of $n = N$ implies that initially all N agents have been given N distinct names, so that all agents have different initial names. This is the same as the symmetric naming game without the invention step. The vocabulary size can be at most N, but in practice, it is much smaller.

In Fig. 6(a) we have plotted the $\langle W_m(n,N)\rangle$ against N for four different values of $n = 2, 3, 4$, and 5, and for six different community sizes N. On a log – log scale the curves fit nicely with straight lines. The averaged overall slope gives a value for $\gamma = 0.99(1)$. Similar plots of $\langle t_m(n,N)\rangle$ and $\langle t_f(n,N)\rangle$ in Fig. 6(b) and Fig. 6(c) give $\alpha = 1.09(1)$ and $\beta = 1.10(1)$ respectively.

Naturally one can ask, what happens if the number n of distinct names grows non-trivially with the number of agents N, e.g., $n = N^{\delta}$? Here $\delta = 0$ corresponds to the case when n has a fixed value, independent of N. On the other hand, $\delta = 1$ implies the case when $n = N$. For other intermediate values of δ in the range $0 < \delta < 1$, simulations have been performed.

Fig. 7(a) exhibits $\langle W_m(\delta,N)\rangle$ against N for $\delta = 0.1, 0.2, \ldots, 1.0$. For each value of δ we get one straight line and the slopes of these lines vary with δ between 1.0 and 1.54. A plot of a total of ten different values of the parameter δ yields different values of $\gamma(\delta)$ and in Fig. 7(b) we plot $\gamma(\delta)$ against δ. Apart from the two limiting points at $\delta = 0$ and 1 the plot

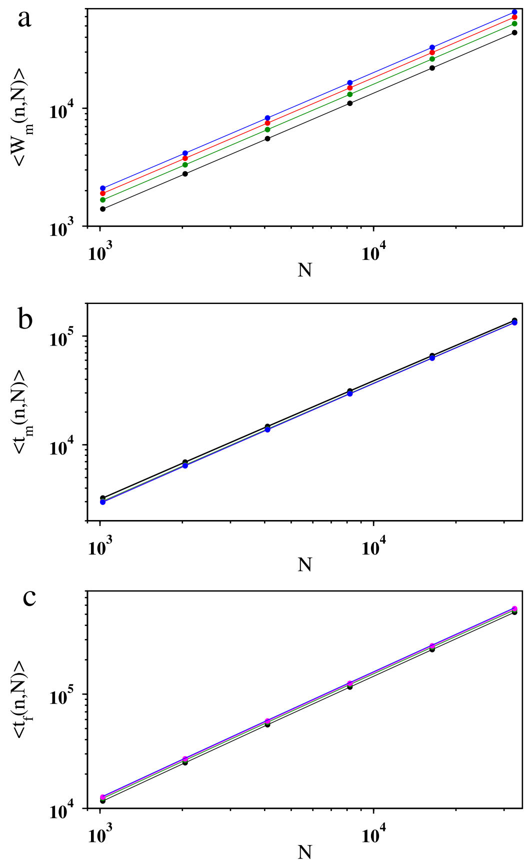

Fig. 6. (a) The configuration averaged value of the maximum number $\langle W_m(n, N)\rangle$ of names has been plotted with the community size N for different constant values of distinct names: $n = 2$ (black), 3 (green), 4 (magenta), 5 (blue). All of them fit to nearly parallel straight lines and have average slope $\gamma = 0.99(1)$. (b) Similar plot for the maximum time $\langle t_m(n, N)\rangle$ against N for same values of n using same colors. The average value of the slopes gives $\alpha = 1.09(1)$. (c) Similar plot for the convergence time $\langle t_f(n, N)\rangle$ against N for same values of n using same colors. The average value of the slopes gives $\beta = 1.10(1)$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

fits nicely to the following relation

$$\gamma(\delta) = 1 + \delta/2. \quad (6)$$

In a similar way we have plotted $\langle t_m(\delta, N)\rangle$ against N for the same values of δ in Fig. 8(a). They almost overlap, yet their slopes $\alpha(\delta)$ vary systematically within a short range. In Fig. 8(b) we have plotted $\alpha(\delta)$ against δ and find it starts from a value $\approx$1.09, then gradually goes through a maximum at $\delta \approx 0.7$, and then decreases to its value $\approx$1.14 at $\delta = 1$.

The plot of the convergence time $\langle t_f(\delta, N)\rangle$ is also quite similar. In Fig. 9(a) we show the variation of this quantity against N, again on a double logarithmic scale. The slopes that yield the values of the exponent $\beta(\delta)$ have been plotted against δ in Fig. 9(b). This plot also shows a mild variation with δ, the maximum occurs around $\delta = 0.7$.

4. Summary

Two modified versions of the symmetric naming game for vocabulary sorting and achieving consensus have been studied with one version exhaustively studied for the asymmetric naming game as well. First, we have introduced a cut-off in the capacity of vocabulary associated with the agents in the community. This modification resulted in a flat plateau in the time variation of the total number $W(t, s, N)$ of names in the community which persists for a long time. The height of the plateau grows linearly with the community size N. For a certain value of the vocabulary size s, there is a crossover for the exponent γ that characterizes the growth of $\langle W_m(s, N)\rangle$ from $\approx$1.0 (strong restriction) with small values of s to $\approx$1.54 with large values of s (weak restriction).

A scaling analysis has been performed that works well. Similarly, the two characteristic time scales, namely, the maximum time t_m and the convergence time t_f also have been seen to obey similar crossover behavior. Our careful analysis yielded that the three exponents characterizing the dynamical evolution of the game have a different set of values not known yet in the literature of naming games.

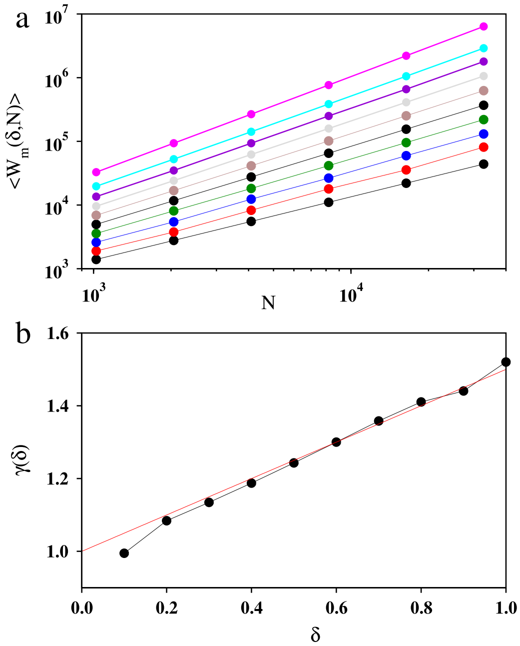

Fig. 7. (Color online) (a) The configuration averaged value of the maximum number $\langle W_m(\delta, N)\rangle$ of names has been plotted with the community size N for different values of the tuning parameter: $\delta = 0.1, 0.2, \ldots, 1.0$ (from bottom to top). Slopes of these lines are the values of the power law exponent γ and are different for different values of δ. (b) Plot of the exponent $\gamma(\delta)$ against δ. Except the two end points, the intermediate region fits well with the form in Eq. (6).

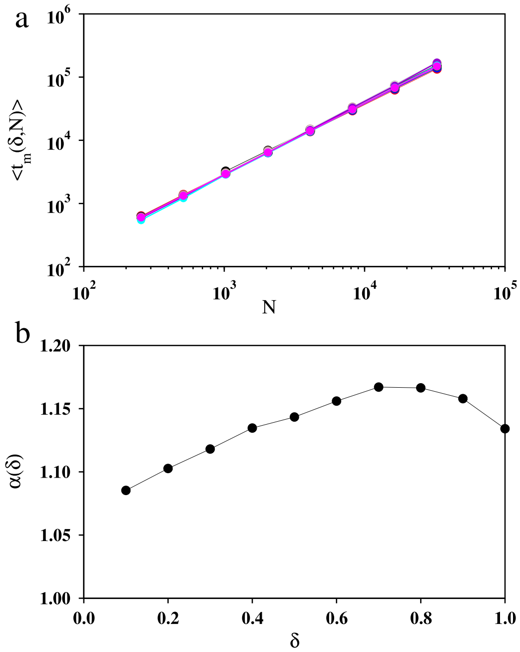

Fig. 8. (Color online) (a) The configuration averaged maximal time $\langle t_m(\delta, N)\rangle$ has been plotted against N using the same values of δ as used in Fig. 7. The slopes measure the value of the exponent α. (b) Variation of the power law exponent $\alpha(\delta)$ against δ.

Effect of a second restriction, imposing a limiting value on the number of distinct names to be assigned to the agents, has been studied. In this case, a constant number n (independent of N) of distinct names has been initially given to the agents. Though this version of the game is much simpler, yet the three characteristic exponents yielded non-trivial values for constant values of n. Further, when we varied n as N^{δ}, the exponents depend non-trivially on the tuning exponent δ.

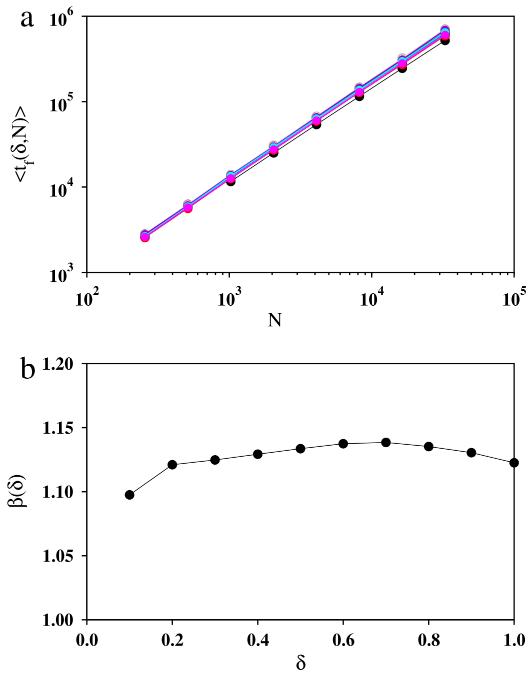

Fig. 9. (Color online) (a) The configuration averaged convergence time $\langle t_f(\delta, N)\rangle$ has been plotted against N using the same values of δ as used in Fig. 7. The slopes measure the value of the exponent β. (b) Variation of the power law exponent $\beta(\delta)$ against δ.

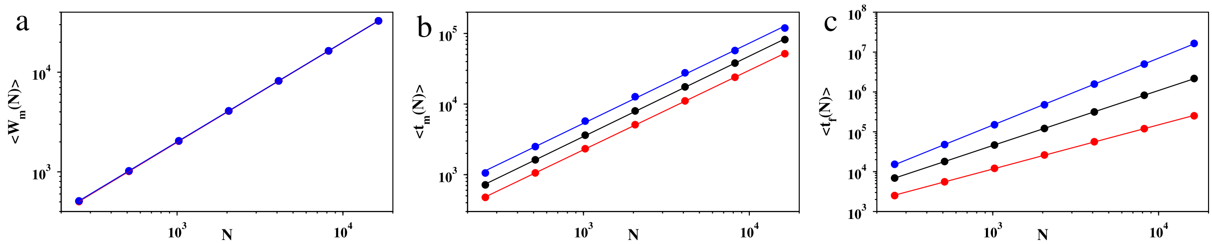

Fig. 10. Data for three different models with $s = 2$ have been presented using three different color symbols: Blue color used for our present model, Black color used for the random selection of names and Red color used for the most successful selection of names. (a) Plot of $\langle W_m(N)\rangle$ against N on log − log scale gives $\gamma = 1.00$, 1.00, and 1.01 respectively. The data for three colors coincide for all values of N. (b) Plot of $\langle t_m(N)\rangle$ against N gives $\alpha = 1.13$, 1.14, and 1.13 respectively. (c) Plot of $\langle t_f(N)\rangle$ against N gives $\beta = 1.61$, 1.38, and 1.11 respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Numerical study of the restricted versions of both the symmetric and asymmetric naming games exhibited non-trivial changes in the asymptotic behaviors of the game. Values of the characterizing exponents have been compared in Table 1 with the asymmetric and the symmetric naming games.

In both the symmetric and asymmetric games, any agent can interact with any other agent. There is no concept of distance between the agents and therefore these games are mean-field models. Yet they have non-trivial values of the exponents for the two models (see Table 1). Here, we like to point out that these exponents are not the usual critical exponents of continuous phase transitions, defined at or near the critical points of phase transitions, exhibiting scale invariance through the finite-size scaling analysis and renormalization group theory. In the naming games, these are only the power law exponents and their values do depend on the different details of the definitions of the models, as described in the following.

How much sensitive are the exponent values with respect to changes in the definition of the model games? To get a feeling for the same, on the suggestions of the reviewer we further studied two modifications of the symmetric naming games with fixed vocabulary sizes at $s = 2$. Only the case of failure in an interaction has been modified in the following way: (i) When $\ell_i + \ell_j > s$, one selects randomly s names from the set of $\ell_1 + \ell_2$ names and after interaction only these s names are copied to the vocabularies of the agents i and j. (ii) A count is kept for the number of successes of every name, and when $\ell_i + \ell_j > s$, only the s top successful names are kept in the vocabularies of the agents i and j. We have found that while γ and α remain same, the value of the exponent β assumes two different values in these models compared to its original value in our model (see Fig. 10). The linguistic implication of these results is both the modifications make the information flow faster. The exponent β is smaller in the random case and has an even smaller value when most successful names are selected implies that the global consensus is reached faster in the first case and fastest in the second case.

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