Colored percolation

Sumanta Kundu and S. S. Manna

Satyendra Nath Bose National Centre for Basic Sciences, Block-JD, Sector-III, Salt Lake, Kolkata-700106, India

A model called "colored percolation" has been introduced with its infinite number of versions in two dimensions. The sites of a regular lattice are randomly occupied with probability p and are then colored by one of the n distinct colors using uniform probability $q = 1/n$. Denoting different colors by the letters of the Roman alphabet, we have studied different versions of the model like $AB, ABC, ABCD, ABCDE, \dots$ etc. Here, only those lattice bonds having two different colored atoms at the ends are defined as connected. The percolation threshold $p_c(n)$ asymptotically converges to its limiting value of p_c as $1/n$. The model has been generalized by introducing a preference towards a subset of colors when m out of n colors are selected with probability q/m each and the rest of the colors are selected with probability $(1-q)/(n-m)$. It has been observed that $p_c(q,m)$ depends nontrivially on q and has a minimum at $q_{\min} = m/n$. In another generalization the fractions of bonds between similarly and dissimilarly colored atoms have been treated as independent parameters. Phase diagrams in this parameter space have been drawn exhibiting percolating and nonpercolating phases.

I. INTRODUCTION

Over the last several decades, the phenomenon of percolation has been proved to be one of the most investigated models in the topic of transport in random disordered systems [1–6]. Broadbent and Hammersley first introduced the model of percolation trying to better understand the mechanism of fluid flow through a random porous medium [7], and now it has become one of the simplest models of studying order-disorder phase transition [8]. Due to its simplicity and plenty of applicability in a number of fields, the literature on this topic is vast, and expectedly a large number of variants of percolation models have been introduced to study the critical behaviors of widely different systems [6,9–16]. In particular, the percolation theory has been successfully applied to the well-known sol-gel transition [17], transitions in conductor-insulator mixtures using random resistor networks [18,19], propagation of fires in forests [1,20], spreading of infectious diseases in the form of epidemics [21,22], etc.

In the ordinary percolation, the sites of a regular lattice are occupied randomly and independently with probability p or kept vacant with probability $(1-p)$. Any two adjacent occupied sites are considered as connected. A group of such occupied sites interconnected through their neighboring connections forms a cluster, the properties of which depend on p. At any arbitrary value of p, there are several clusters of different shapes and sizes. The size of the largest cluster increases monotonically as p is increased, and right above a critical value of $p = p_c$, known as the percolation threshold, the largest cluster includes sites on the opposite sides of the lattice and thus for the first time a global connectivity is established. Therefore, p_c marks the transition point, between the globally connected and disconnected phases, characterized by the divergence of the correlation length $\xi(p)$ as $p \to p_c$. It is well known that the ordinary percolation undergoes a continuous phase transition at $p = p_c$, and the set of critical exponents defined at and around p_c characterizes the universality class of the transition [1]. The best known value of p_c(sq) for the site percolation on the square lattice is 0.59274605079210(2) [23] and 1/2 for the bond percolation [24].

Inspired by the phenomena of anti-ferromagnetism, gelation, spreading of infection from the infected cells to normal cells, Mai and Halley introduced the AB percolation model [2,25–27]. The model of AB percolation is illustrated in the following way. Initially, all sites of a lattice are occupied with B atoms. Then, random sites are selected one by one and the B atoms at these sites are replaced by the A atoms. At any arbitrary intermediate stage the fraction of A atoms is denoted by r. According to this model, bonds which are having both A and B atoms at their opposite ends are marked as connected. For a given value of r, the probability that any given edge has a bond is $2r(1-r)$, which has its maximum at $r = 1/2$ and decreases monotonically on both sides of this point. Consequently, the average size of the largest cluster gradually grows till $r = 1/2$. The entire scenario is symmetric about $r = 1/2$. At $r = 1 - r_c$, the size of the largest cluster drops sharply, and finally it vanishes at $r = 1$. Therefore, for $r_c \leqslant r \leqslant 1 - r_c$, the system is percolating. However, the existence of a global connectivity through the alternating A and B atoms, and therefore the existence of r_c, crucially depends on the geometry of the underlying lattice [27,28]. For example, the spanning AB cluster does not exist on the square lattice [28,29], but it exists on the triangular lattice [30]. Although it was first concluded that the universality class of AB percolation in two dimensions is different from the ordinary percolation [26], later it has been argued that it belongs to the same universality class as the ordinary percolation [31–33]. Further, random occupation of lattice sites by more than two distinct atoms was studied through the model of polychromatic percolation [34,35].

In this paper, we consider a percolation model, where the sites of a regular lattice are occupied with probability p similar to the ordinary site percolation, and then at every occupied site one of the n differently colored atoms is assigned with a given probability q. A bond between a pair of neighboring occupied sites is declared as connected if the atoms are of different colors. We refer to this model as colored percolation, and we study the critical properties of this model for both the square and triangular lattices.