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# Intermittent granular flow and clogging with internal avalanches

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**Abstract.** The dynamics of intermittent granular flow through an orifice at the bottom of a granular bin and the associated clogging due to formation of arches blocking the outlet, is studied numerically in two dimensions. When the hole size is less than the grain diameter, only a single grain is removed from the system so that the system self-organizes to a steady state and the distribution of the grain displacements decays as power laws. On the other hand, when hole sizes are within few times of the grain diameter, the outflow distributions are also observed to follow a power law.

**PACS.** 05.70.Jk Critical point phenomena – 74.80.Bj Granular, melt-textured, and amorphous superconductors; powders

How much granular mass is expected to flow out through a hole at the bottom of a granular bin? If the hole is sufficiently small, the outflow of grains stops soon due to the formation of a stable arch of grains clogging the hole. If the arch is broken, the grains in the arch become unstable and a cascade of grain displacements propagates within the system called the “internal avalanche”. As a result, a fresh flow starts through the hole, which also eventually stops due to the formation of another arch. It is known that the amount of outflowing granular mass between successive clogging events varies over a wide range, however, on average it depends sensitively on the hole diameter as compared to the grain size.

In this paper, we study this phenomenon of clogged outflow distribution from a granular bin in two dimensions using computer simulations. We observe the power law dependence of the distribution.

This phenomenon is important in the context of recent experimental and theoretical studies of the avalanche size distributions from the surfaces of a sandpile on an open base [1–5, 7]. Research in this direction is motivated by the expectation that a sandpile is a simple example of a Self-Organized Critical (SOC) system [1–3]. According to the ideas of SOC, long range spatio-temporal correlations may spontaneously emerge in slowly driven dynamical systems without fine tuning of any control parameter [1].

Laboratory experiments on sandpiles, however, have not always found evidence of criticality in sandpiles. The fluctuating outflow of granular mass from a slowly rotating horizontal semicircular drum did not show a power law decay of its power spectrum as expected from the SOC

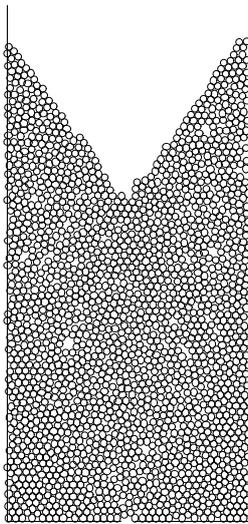
theory [4]. In a second experiment, the avalanche size was directly measured by the outflow amount during the dropping of sand on a sandpile situated on a horizontal base. It was observed that the distribution of the avalanche size obeys a scaling behaviour only for small piles but not for the large ones [5]. However, it was claimed that a stretched exponential distribution for the avalanches fits the whole range of system sizes [6]. The reasons for not observing scaling are the existence of two angles of repose as observed in [4] and also the effect of inertia of the sand mass while moving down during avalanches. This effect was suppressed in an experiment with anisotropic rice grains and scaling was successfully observed [7].

In contrast, we believe, internal avalanches are better candidates for observing SOC. Due to the high compactness of grains in the granular material in a bin, a grain never gets sufficient time to accelerate much and the effect of inertia should be small. Internal avalanches were first studied in a block model of granular system by removing grains situated at the bottom of the bin one after another [8]. While this model and also other cellular automata [9] and “Tetris-like” models [10] did exhibit scaling behaviour for the internal avalanches, the disc models [9, 11, 12] in continuous space, however, did not show sufficient evidence of SOC.

We study a two-dimensional granular system with grains kept in a vertical rectangular bin. Initially the system has no hole at the bottom and we fill the bin up to a certain height. A hole of half-width  $R$  is then made at the centre of the bottom. Some grains drop out through the hole. This flow is stopped when an “arch” is formed, clogging the flow (Fig. 1). In two dimensions, an arch is a sequence of grains, mutually depending on one another,

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**Fig. 1.** A granular system of 2000 grains of radii  $1/\sqrt{2}$  unit in a bin of width 50 units. The bin has a hole of width 3 units at the centre. The figure shows the steady state of the system after 3000 avalanches.

such that their weights are balanced by mutual reaction forces. These arches are formed due to the competition among the grains to occupy the same vacant region of space. In the steady state, these arches are observed everywhere. The arch, covering the hole, is broken by deleting randomly one of the two grains at the bottom of the arch. An avalanche of grain movements starts within the system. Those grains which reach the bottom line with their centres within the hole are considered to flow out of the system and are deleted. A deleted grain is immediately replaced again randomly on the top surface. We measure the number of grains  $\rho$  that flow out of the system during an avalanche between two successive clogging events and study its distributions for various values of the hole sizes  $R$ .

The granular system is represented by  $N$  hard monodisperse discs of radii  $r$ . No two grains are allowed to overlap but they can touch each other and one can roll over the other if possible. A rectangular area of size  $L_x \times L_y$  on the  $x$ - $y$  plane represents our two-dimensional bin. Periodic boundary conditions are used along the  $x$  direction and gravity acts along the  $-y$  direction.

A “pseudo-dynamics” [9] method is used to study the avalanches. In this method we do not solve the classical equations of motion for the grain system as should have been done in a Molecular Dynamics method. Here, the direction of gravity and the local geometrical constraints due to the presence of other grains govern the movement of a grain. To justify the use of the pseudo-dynamics, we argue that due to the high compactness of the system a grain never gets sufficient time to accelerate to large velocities. During the avalanches, the grains move in parallel. This is mimicked in the simulation by discretizing the time. In one time unit, all grains are given one chance each to move, by selecting them one after another in any random sequence. A grain can move only in two ways: either it can

*fall* a certain distance or *roll* a certain angle over another grain. We used only one parameter  $\delta$  to characterize these two cases. Therefore  $\delta$  is the maximum possible free fall a grain can have in a unit time. While rolling, the centre of a grain moves along the arc of a circle of radius  $2r$  on another grain in contact, the maximum length of the arc is also limited to  $\delta$ . Therefore, the maximum angle a grain can roll freely is  $\theta = \delta/2r$  over another grain in contact. We believe this dynamics models an assembly of very light grains in 0+ gravity.

The presence of other grains in the neighbourhood of a particular grain imposes severe restriction to its possible movements. This is taken care of in the simulation using a search algorithm which digitizes the bin into a square grid and associates the serial numbers of grains to the primitive cells of the grid. A cell contains at most one grain when  $r \geq 1/\sqrt{2}$  is chosen. Then it needs to search only 24 nearest neighbouring cells for possible contact grains. A grain  $n$  in the stable position is supported by two other grains with serial numbers  $n_L$  and  $n_R$  on the left (L) and right (R). The position of the grain  $n$  is then updated according to the following prescription:

- if  $n_L = n_R = 0$ , it falls,
- if  $n_L \neq 0$  but  $n_R = 0$ , it rolls over  $n_L$ ,
- if  $n_L = 0$  but  $n_R \neq 0$ , it rolls over  $n_R$ ,
- if  $n_L \neq 0$  and  $n_R \neq 0$ , it is stable.

When the grain  $n$  with the centre at  $(x_n, y_n)$  is allowed to fall, it may come into contact with a grain  $m$  at  $(x_m, y_m)$  within the distance  $\delta$ . Therefore, during the fall, the coordinates are updated as:

$$x'_n = x_n \quad \text{and} \quad y'_n = y_m + \sqrt{4r^2 - (x_n - x_m)^2}$$

$$\text{if } y_n - y'_n < \delta, \quad \text{otherwise } y'_n = y_n - \delta.$$

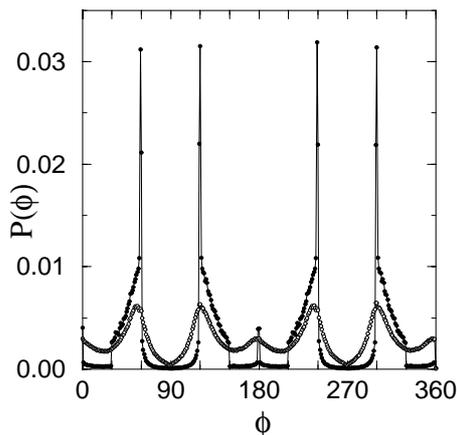
Similarly the grain  $n$  may roll an angle  $\theta'$  over a grain  $t$  and come into contact with another grain  $m$  at  $(x_m, y_m)$ . The coordinates of the grain  $n$  in the new position where it touches both  $t$  and  $m$  are:

$$x'_n = \frac{1}{2}(x_m + x_t) \pm (y_m - y_t) \sqrt{\frac{4r^2}{d_{mt}^2} - \frac{1}{4}},$$

$$y'_n = \frac{1}{2}(y_m + y_t) \mp (x_m - x_t) \sqrt{\frac{4r^2}{d_{mt}^2} - \frac{1}{4}}.$$

Here  $d_{mt}$  is the distance between the centres of the grains  $m$  and  $t$  and the  $\pm$  signs are for the left and right rolls. This position is accepted only if  $\theta' \leq \theta$  otherwise the grain  $n$  rolls down a maximum angle  $\theta$  over the grain  $t$ .

The initial configuration of grains within the bin is done by releasing grains sequentially one after another along vertical trajectories with randomly chosen horizontal positions. They fall till they touch the growing heap and are then allowed to roll down on the surface to their local stable positions. This is called the “ballistic deposition with re-structuring method” (BDRM) [13]. The initial pattern does not have arches since a grain while rolling down along the surface does not need to compete with any

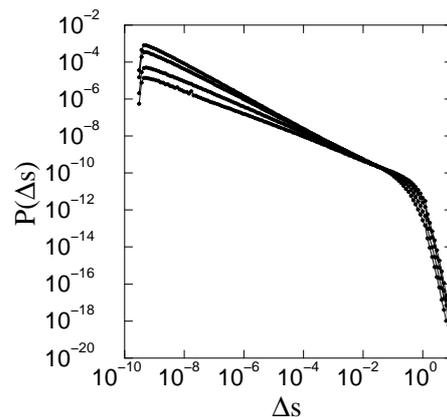


**Fig. 2.** Distribution  $P(\phi)$  of the angles  $\phi$  of the contact vectors. The plot for the initial distribution is shown by black dots. It has four high peaks at  $60^\circ$ ,  $120^\circ$ ,  $240^\circ$ ,  $300^\circ$ . The curve with opaque circles corresponds to the steady state. The randomness introduced into the system is reflected by the drastic reduction of the height of the peaks as well as a broadening.

other grain. However, the arches are observed when the system is disturbed by opening the outlet and allowing the grains to flow out.

After a large number of outflows, the system reaches a steady state. One can characterize this state in many ways. We characterize the steady state by the distribution of the angles of the contact vectors. When two grains are in contact, we draw two vectors from their centres to the contact points. The angle  $\phi$  is the angle between the contact vector and the  $+x$  direction. We measured the probability distributions  $P(\phi)$  in the range  $0-360^\circ$  for the initial state as well as in the steady state. To arrive at the steady state we discard  $2N$  outflows, though the transition takes place even earlier than that.

The contact angle distributions are shown in Figure 2. The plot of  $P(\phi)$  vs.  $\phi$  for the initial grain distribution shows four sharp peaks at  $60^\circ$ ,  $120^\circ$ ,  $240^\circ$ ,  $300^\circ$ . We explain them in the following way. If the grains were placed at the bottom in a complete orderly way by mutually touching one another without any gap, the structure generated by the BDRM method would be the hexagonal close packing (HCP) structure where every grain has six other grains in contact at the interval of  $60^\circ$ . However, in our case, though we dropped grains randomly at the bottom level, the deterministic piling process retains some effect of the HCP structure. On the other hand, it is also known that the probability of a grain having the number of contact neighbours different from four, decreases with the height as a power law [13], which is certainly consistent with the four peaks we obtained. There are also two small peaks at  $0^\circ$  and at  $180^\circ$  due to the horizontal contacts of grains near the base level. This distribution, however, changes drastically by reducing the heights of the peaks and by broadening their widths in the steady state. The randomness introduced into the system by randomly selecting grains at the bottom increases the possibilities of

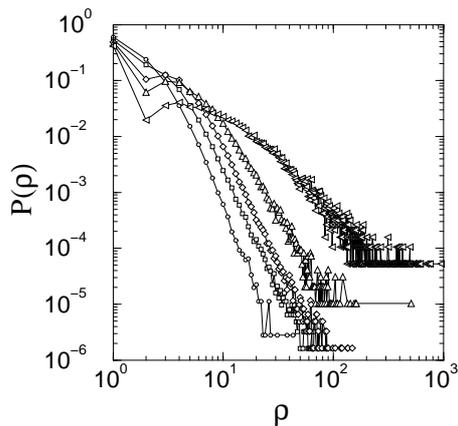


**Fig. 3.** Distribution  $P(\Delta s)$  of the displacement  $\Delta s$  of the grains in a granular system in the limit of  $R \rightarrow 0$ . Curves for different sizes with grain numbers  $N = 300$ ,  $625$ ,  $2500$  and  $10000$  are plotted from bottom to top on the left.

other contact angles also, but is not fully capable of totally randomizing it to a flat distribution. We check that after a large number of avalanches, this steady state distribution remains unchanged.

Next we study the statistics of the displacement distributions in the granular heap in the limit of  $R \rightarrow r$ . In this limit, no grain can drop out through the hole. The arch is broken by deleting only one grain. The avalanche is allowed to propagate within the system. The displacement  $\Delta s$  is the absolute magnitude of the displacement of a grain before and after an avalanche. We observe that there is a huge variation in the displacements  $\Delta s$  of the grains. Most of the grains displace very little, whereas others have much bigger displacements, but their numbers are small. The displacement distribution  $P(\Delta s)$  is measured in the steady state over a large number of avalanches. The lower cut-off of the distribution turned out to be strongly dependent on the tolerance factor  $\epsilon$  used for the simulation. If the centres of two grains are separated by a distance within  $r - \epsilon$  and  $r + \epsilon$ , then they are considered to be in contact. The upper cut-off of the distribution is of the order of unity since when a grain is deleted at the bottom, its neighbouring grains drop a distance of the order of the grain diameter.

Systems of four different sizes have been simulated with  $N = 300$ ,  $625$ ,  $2500$  and  $10000$  in bins with base sizes  $L_x = 20$ ,  $40$ ,  $80$  and  $160$  units, respectively. First  $5N$  avalanches are thrown away to allow the system to reach the steady state. A list is maintained for the coordinates of all grains. This list is compared before and after the avalanche to measure  $\Delta s$ . We observe a nice straight line curve over many decades of the  $P(\Delta s)$  vs.  $\Delta s$  plot on a double logarithmic scale, with little curvature at the right edge (Fig. 3). However, the slope of this curve is found to increase with the system size. Different extrapolations are tried and the best is obtained with  $L^{-1/2}$  which leads to  $0.97 \pm 0.05$  in the  $N \rightarrow \infty$  limit for the slope. We



**Fig. 4.** The distribution  $P(\rho)$  of the number of grains  $\rho$  that flows out of the system in an internal avalanche between two successive clogged situations plotted on a double logarithmic scale. A bin of size  $L_x = 50$ , containing 4000 grains of radii  $r = 1/\sqrt{2}$  was simulated with the hole sizes  $R = 0.75, 1, 1.25, 1.5$ , and  $2.0$  (curves from left to right).

get similar behaviours for the distributions of the  $x$  and  $y$  components of the displacement vectors.

Finally we study the outflow distribution and clogging. We use a bin of dimensions  $L_x = 50$  and  $L_y = 200$  units containing 4000 grains of radii  $r = 1/\sqrt{2}$  with a hole of half-width  $R$  at the centre of the bottom of the bin. In a stable state an arch covers the hole and forbids other grains to flow out. The two lowermost grains of the arch on the two sides of the hole are chosen and one of them is selected randomly. By deleting this grain, the arch is made unstable, which creates an internal avalanche of grain movements within the bin. Grains which come down and touch the bottom line of the bin with their centres within the hole are removed. While removing such a grain, the whole avalanche dynamics is frozen and the grain is replaced back on the top surface again by the BDRM method.

Two conical shapes are formed. There are some grains which are never disturbed by any avalanche and are located on both sides of the hole. These undisturbed grains form a cone. The second cone is formed on the upper surface, though the grains were dropped on this surface along randomly chosen horizontal coordinates with uniform probability. For a fixed size of the base, the angle of the cone is found to increase with the average height of the granular column.

The number of grains  $\rho$  that go out of the system in an internal avalanche is counted between successive clogging events. Holes of five different sizes, *i.e.*,  $R = 0.75, 1, 1.25, 1.5, 2$  are used. We could generate around  $5 \times 10^5$  such outflows for each hole size and collected the distribution data for  $P(\rho)$ . We show the plot of these curves in Figure 4 on a double logarithmic scale. The grain which is deleted to break the arch is also counted in  $\rho$ . It is found increasingly improbable that the outflow will consist of only another grain. Consequently  $P(1)$  is found

to decrease monotonically on increasing  $R$ . However, for large values of  $\rho$ , the curve falls down.

All curves of  $P(\rho)$  vs.  $\rho$  plots show straight lines for large  $\rho$  values, signifying the power law distributions for the avalanche sizes. The slopes of these curves  $\sigma(R)$  are found to depend strongly on the size of the hole, as well as the grain size:  $P(\rho) \sim \rho^{-\sigma(R,r)}$ . We observe that  $\sigma(R,r)$  is actually a linear function of  $(R/r)$  as  $\sigma(R,r) = k(R/r)$  with  $k = 1.6$ . However, since our system size is small and also the sample size, the continuous variation of the  $\sigma(R,r)$  may just be due to the corrections to scaling as one moves away from the critical point. The average outflow  $\langle \rho(R/r) \rangle$  diverges exponentially as:  $\exp(\alpha(R/r - 1))$  with  $\alpha = 0.7$ .

To summarize, we have studied a numerical model of intermittent granular flow and clogging with associated internal avalanches from a two-dimensional bin using a pseudodynamics method. The flow of grains through a hole at the bottom of the bin is repeatedly clogged due to the formation of stable arches of grains, which we break to start fresh flows. Our numerical simulation results indicate that the system self-organizes to a state different from the initial state. In the limit of  $R \leq r$ , the distribution of the granular displacements follow power laws. On the other hand, when  $R > r$ , the granular flow distribution is also observed to follow a power law.

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