

From brane assisted inflation to quintessence through a single scalar field

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We explore within the context of brane cosmology whether it is possible to obtain both early inflation and accelerated expansion during the present epoch through the dynamics of the same scalar field in an exponential potential. Considerations from successful inflation and viable radiation and matter dominated eras impose constraints on the parameters of the potential. We find that the additional requirement of late time quintessence behavior in conformity with present observations necessitates the inclusion of two exponential terms in the potential.

PACS number(s): 98.80.Cq

I. INTRODUCTION

The idea that our universe is a brane embedded in higher dimensional space has received considerable attention in recent years [1]. This has been motivated by solutions of string theory where all matter and gauge fields are confined to the 3-brane, whereas gravity can propagate in the bulk. In these schemes the extra dimensions need not be small or compact, a radical departure from the standard Kaluza-Klein scenario, and the fundamental Planck scale could be significantly smaller than our effective four dimensional Planck scale m_{pl} [2]. A lot of effort is currently being devoted to understand the cosmology of such a brane world scenario [3].

The most important feature that distinguishes brane cosmology from the standard scenario is the fact that at high energies the Friedmann equation is modified by an extra term quadratic in energy density ρ [3]. It is expected that the implications of such a modification would be profound for the inflationary paradigm. Recent measurements of the power spectrum of cosmic microwave background (CMB) anisotropy [4] provide a strong justification for inflation [5]. It has been realized that the brane world scenario is more suitable for inflation with steep potentials because the quadratic term in ρ increases friction in the inflaton field equation [6]. This feature has been exploited to construct inflationary models using both large inverse power law [7] as well as steep exponential [8] potentials for the scalar field.

A common ingredient in these models is that reheating is supposed to take place through gravitational particle production. The conventional reheating mechanism through decay of the inflaton cannot be implemented in these models because the scalar field potential does not have a minimum near the scale of inflation. At the end of inflation, the scalar field equation becomes kinetic energy dominated because of the steepness of the potential. This energy is redshifted rapidly and radiation domination ensues. The detailed aspects of the scenario of gravitational particle production have been debated in Refs. [9]. The condition that radiation domination sets in before nucleosynthesis is used to impose constraints on the parameters of brane inflation models [7,8]. A definite prediction of these models is the parameter independence of

the spectral index of scalar density perturbations [6–8].

The continued rolling down the potential slope of the inflaton field raises interesting questions about its late time behavior. Preliminary analyses support the idea that the same scalar field can provide inflation at early times, and behave as a quintessence field at late times [7,8,10]. Recent observations of distant supernovae and galaxy clusters seem to suggest that our universe is presently undergoing a phase of accelerated expansion [11], indicating the dominance of dark energy with negative pressure in our present universe. The idea that a slowly rolling scalar field provides the dominant contribution to the present energy density has gained prominence in recent times [12]. The crucial feature for viability of this theme is the existence of late time attractor solutions for a wide range of initial conditions for certain types of potentials [13]. The possibility of obtaining quintessence through tracker-type potentials has been analyzed through a variety of power law or exponential potentials, and also combinations of these [14], though a recent study [15] seems to disfavor models with large inverse power law potentials.

In this paper we consider a potential consisting of a general combination of two exponential terms given by

$$V(\phi) = V_0[A \exp(-\alpha\phi/m_{pl}) + B \exp(-\beta\phi/m_{pl})]. \quad (1)$$

Such a potential has been recently claimed [16] to conform to all the observational constraints to date on quintessence for certain values of parameters. Our goal is to check in detail whether a viable scenario of brane world inflation, and a feasible late time quintessence solution both work out with this potential. We impose all the requisite constraints that come into play in inflation, during the intermediate radiation and matter dominated eras, and finally from the present epoch of accelerated expansion. Balancing the dynamics between such disparate scales requires, as expected, a tuning of one parameter in the potential. This is just a rephrasing of the cosmological constant problem which cannot be addressed through this mechanism. Nonetheless, our results indicate that such a unified scheme is possible with certain restrictions on the allowed values for the potential parameters. Our analysis serves to highlight the inevitability of requiring two exponential terms and also to rule out certain specific categories (formed by the selection of particular combinations of values and signatures of parameters) of the above potential.

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II. INFLATION IN THE BRANE WORLD SCENARIO

To begin with, let us note certain points regarding the parameters in potential (1). First, one of the constants A or B can be absorbed in V_0 . Secondly, $V(\phi)$ is invariant under the transformation $\alpha \rightarrow -\alpha$, $\beta \rightarrow -\beta$, $\phi \rightarrow -\phi$. So without loss of generality we can set $B=1$, $\alpha < 0$, and $\phi_i > 0$, where ϕ_i is some initial value of ϕ . Further, we assume that either of the two conditions, (a) $-\alpha \gg |\beta|$, or (b) $|A| \gg 1$, for $\alpha/\beta \approx +1$ hold. This makes the first term in $V(\phi)$ dominate except for very small or negative values of ϕ , thereby enabling us to make use of the analysis of Copeland *et al.* [8] during the early inflationary era.

The brane world dynamics is governed by the modified Friedmann equation

$$H^2 = \frac{8\pi}{3m_{pl}^2} \rho \left(1 + \frac{\rho}{2\lambda} \right) \quad (2)$$

where, for reasons of simplicity, we have set the contributions from bulk gravitons and higher dimensional cosmological constant to zero [6–8]. The brane tension λ defines the scale below which the quadratic correction begins to lose importance and standard Friedmann evolution is recovered. The scalar field obeys the equation

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0. \quad (3)$$

As long as the condition $\rho/2\lambda > 1$ holds, the quadratic term in Eq. (2) dominates and contributes to increased friction in the scalar field equation (3). Thus during inflation, one can define a modified slow roll parameter

$$\epsilon(\phi) \equiv \frac{m_{pl}^2}{8\pi} \left(\frac{V'(\phi)}{V(\phi)} \right)^2 \frac{2\lambda}{V(\phi) + 2\lambda}. \quad (4)$$

If the brane energy momentum tensor is dominated by the energy density of the scalar field, inflation ensues when $\epsilon < 1$. The condition for brane assisted inflation¹ is thus weakened compared to the standard case, as here $p < -\frac{2}{3}\rho$ guarantees accelerated expansion, i.e., $\ddot{a} > 0$. In the slow roll expansion, the number of e-foldings N is given by

$$N \approx - \frac{8\pi}{m_{pl}^2} \int_{\phi_i}^{\phi_e} \frac{V(\phi)}{V'(\phi)} \left(\frac{2\lambda + V(\phi)}{2\lambda} \right) d\phi \quad (5)$$

where ϕ_e denotes the value of ϕ at the end of inflation ($\epsilon \approx 1$). Assuming $V \gg \lambda$ during inflation, one obtains

$$V_0 \approx \frac{\lambda \alpha^2}{4\pi A} \exp\left(\frac{\alpha \phi_e}{m_{pl}}\right) \quad (6)$$

¹The term ‘‘assisted inflation’’ was coined in the context of standard Friedmann cosmology where cumulative effects of multiple scalar fields in exponential potentials give rise to inflation [17]. However, in this paper we use the term ‘‘brane assisted inflation’’ simply to signify the effect of the quadratic energy density correction towards helping slow roll.

and

$$V_e \approx \frac{\lambda \alpha^2}{4\pi}. \quad (7)$$

Hence, the number of e-foldings N can be written as

$$N \approx \exp\left(\frac{-\alpha(\phi_i - \phi_e)}{m_{pl}}\right) - 1 \approx \frac{4\pi}{\lambda \alpha^2} (V_N - V_e), \quad (8)$$

where the value of the potential at N e-foldings from the end of inflation is given by

$$V_N = V_e(N+1). \quad (9)$$

Following Refs. [6,8] one can define the amplitude of density perturbations as

$$A_s^2 \approx \frac{64\pi V^4(\phi)}{75\alpha^2 \lambda^3 m_{pl}^4}. \quad (10)$$

Using the Cosmic Background Explorer (COBE) normalization $A_s = 2 \times 10^{-5}$ [18], one gets

$$\lambda \approx \left(\frac{10^{15} \text{ GeV}}{(-\alpha/\sqrt{8\pi})^{3/2}} \right)^4. \quad (11)$$

Though we have assumed $V(\phi) \gg \lambda$ during inflation, consistency demands that the scalar field be confined to the brane at all times. One has to ensure that $V_{max} < (M_5)^4$, where M_5 is the five dimensional Planck scale, related to m_{pl} and brane tension λ by

$$m_{pl} = \sqrt{\frac{3}{4\pi}} \left(\frac{(M_5)^2}{\sqrt{\lambda}} \right) M_5. \quad (12)$$

The above consistency condition [$V_{max} < (M_5)^4$] leads to the constraint

$$\frac{-\alpha(\phi_{max} - \phi_e)}{m_{pl}} \leq 13.8. \quad (13)$$

Using Eqs. (8) and (13) one can see that the maximum number of e-foldings possible is given by

$$N_{max} \approx \exp\left(\frac{-\alpha(\phi_{max} - \phi_e)}{m_{pl}}\right) \approx 10^5. \quad (14)$$

We recover the results of brane assisted inflation presented in Ref. [8] since our model reduces to theirs during inflation. It should be mentioned here that two generic predictions of brane inflation (the spectral index $n_s \approx 0.92$, and the ratio of the amplitude of tensor to scalar perturbations, $A_T^2/A_S^2 \approx 0.03$), are independent of the slope α of the exponential potential [6,8,19]. However, to produce an observationally acceptable value for n_s and A_T^2/A_S^2 , one needs a very high power of ϕ for inverse power law potentials [7].

III. CONSTRAINTS FROM RADIATION AND MATTER DOMINATED ERAS

The scalar field potential at the end of inflation is given by V_e [Eq. (7)]. At this stage the brane correction to Friedmann equation is dominant for $\alpha^2 > 8\pi$. The field ϕ continues to roll down in the absence of any minimum for the potential at this scale. Thus reheating can take place only through gravitational particle production [9]. Assuming this to be the case, the energy density in radiation at the end of inflation is given by

$$(\rho_R)_e \approx g \left(\frac{10^{11} \text{ GeV}}{-\alpha/\sqrt{8\pi}} \right)^4 \quad (15)$$

where g is the number of fields which produce particles at this stage. The ratio of radiation density to scalar field density is $(\rho_R)_e/(\rho_\phi)_e \approx g(10^{-17})$ [8]. Radiation red shifts as $\rho_R \propto a^{-4}$ and competes with the scalar field energy density ρ_ϕ for domination. The equation state for ϕ after inflation is $\omega_\phi \geq -2/3$. ρ_ϕ falls off starting from $\rho_\phi \propto a^{-1}$ (when brane effects are most important) to $\rho_\phi \propto a^{-6}$ (complete kinetic domination). In the rather unlikely case of the former evolution throughout the time until $\rho < 2\lambda$, the onset of radiation domination gets delayed. Radiation domination ensues after the scale factor has expanded by an extra factor of $(V_e/2\lambda)^{5/3} \approx (-\alpha/\sqrt{8\pi})^{10/3}$ [8] over and above the factor of around 10^7 by which it would have expanded had kinetic domination of the scalar field set in immediately at the end of inflation. The temperature of radiation at the onset of radiation domination is given in this scenario by

$$T_{RD} \approx \frac{10^4 \text{ GeV}}{(-\alpha/\sqrt{8\pi})^{13/3}}. \quad (16)$$

Radiation density in the universe must dominate before nucleosynthesis, i.e., $T_{RD} \geq 1 \text{ MeV}$. Hence, one obtains an upper bound on α , i.e., $\alpha \leq 10^2$. In practice, however, this scenario is extremely unlikely because the steep nature of the potential will quickly force ρ_ϕ to be dominated by kinetic energy ($\omega_\phi \approx 1$, $\dot{\phi} = a^{-3}$), and the above bound should be interpreted in a loose sense. Keeping this caveat in mind, one could still calculate the number of e-foldings encountered by the ϕ field from the end of inflation to nucleosynthesis, which is given by

$$\exp\left(\frac{-\alpha(\phi_e - \phi_{nuc})}{m_{pl}}\right)\Bigg|_{K.D.} \approx \frac{10^{21} \alpha^{10}}{(8\pi)^5}. \quad (17)$$

In the more likely case of brane effects continuing to play a role in the dynamics for some time after the end of inflation, a more plausible solution for ϕ is of the ‘‘tracker type’’ [13] where $\omega_\phi \approx \omega_r = 1/3$. In this case it turns out that

$$\exp\left(\frac{-\alpha(\phi_e - \phi_{nuc})}{m_{pl}}\right)\Bigg|_{tracker} \approx \left(\frac{a_{nuc}}{a_e}\right)^2. \quad (18)$$

Before proceeding further, one needs to ensure that the temperature of radiation density at the end of inflation given by Eq. (16) does not exceed 10^9 GeV , the temperature at which thermal production of gravitinos might be significant [20]. It can be checked from Eq. (16) that $T_e \leq 10^9 \text{ GeV}$ implies $-\alpha \geq 10^2$. Using this bound together with the requirement for the onset of radiation domination before nucleosynthesis, we set the value of α to be

$$-\alpha \approx O(10^2). \quad (19)$$

Recently, it has been claimed that the requirement that reheat temperature after inflation be bounded by production of excess gravitons limits the brane tension, i.e., $\lambda \geq 10^{11} \text{ GeV}^4$ [21]. Our choice of α is consistent with this bound.

Exponential potentials are known to yield tracker solutions. Here we look for a late time attractor or scaling solution where the scalar field dynamics mimics that of the dominant background fluid (radiation or matter) with an approximately constant ratio between their energy densities. The tracking condition [13] requires that

$$\frac{V'(\phi)}{V(\phi)} \approx \frac{m_{pl}^{-1}}{\sqrt{\Omega_\phi}} \quad (20)$$

holds at various stages of evolution. Equation (20) can be used to check the consistency of the constraint (19) at different stages, for example, at t_{nuc} when the recently obtained bound $(\Omega_\phi)_{nuc} < 0.045$ [22] needs to be satisfied. The existence of tracking behavior can be determined by the quantity [13]

$$\Gamma \equiv \frac{V''(\phi)V(\phi)}{(V'(\phi))^2}. \quad (21)$$

If Γ stays nearly constant, then a solution converges to a tracking one. For our model it is easy to verify that $\Gamma = 1$ whenever either of the two terms dominate in Eq. (1). [With our choice of conditions (a) or (b), we want the first term to dominate during inflation, as well as throughout the radiation and matter dominated eras. The transient regime when both the terms play equitable roles in the dynamics will be discussed in the next section.]

The exact evolution of the ϕ field will depend on its energy fraction Ω_ϕ and its equation of state parameter ω_ϕ which varies with time all the way from $\omega_\phi \approx 1$ (kinetic domination) at some stage after inflation to $\omega_\phi \rightarrow -1$ (quintessence) during the present epoch. Analysis of recent CMB data constrains $\Omega_\phi \leq 0.39$ during the radiation dominated epoch [22]. Numerical analysis [23] suggests that from t_{nuc} to t_{eq} Ω_ϕ rises slowly staying nearly constant around 0.2. Assuming ω_ϕ tracks the behavior of radiation (i.e., $\omega_\phi \sim 1/3$) up to the era of matter-radiation equality, one obtains

$$\exp\left(\frac{-\alpha(\phi_{nuc} - \phi_{eq})}{m_{pl}}\right) \approx \left(\frac{a_{eq}}{a_{nuc}}\right)^2. \quad (22)$$

From t_{eq} to the present era the field ϕ experiences a few more e-foldings ($\exp[-\alpha(\phi_{eq} - \phi_{now})/m_{pl}] \approx O(1)$). Dur-

ing galaxy formation $\Omega_\phi \leq 0.5$, and hence, the dynamics should be such that $-0.5 < \omega_\phi < -1/3$ during this era [23]. These are approximate results in the sense that for more accurate results one has to solve the equations of motion for ϕ and H . Nevertheless, our use of average values for ω_ϕ during different eras is justified as our purpose here is to estimate the available parameter space for ϕ before Ω_ϕ domination sets in around the present epoch. Or, in other words we estimate the total magnitude of rolling down the effective slope α of the potential experienced by ϕ until the present phase of accelerated expansion influenced by the second term in Eq. (1) is arrived at.

IV. ACCELERATED EXPANSION DURING THE PRESENT EPOCH

With a high level of confidence present observations suggest that our universe has entered an era of accelerated expansion driven by a cosmological constant or energy density of a scalar field with $\Omega_\phi \approx 0.7$ [11,24]. Our model with effective slope α describing the dynamics from inflation to matter domination is unable to account for a second period of accelerated expansion. This is because the late time attractor solution used here has $\omega_\phi \approx \omega_m$. Although recently it has been claimed that a viable model of quintessence could work with a potential having a single exponential term [25], the allowed value for the slope of the potential is two orders of magnitude smaller than the value of α that needs to be used by us. It is known that exponential potentials also allow for another kind of late time attractor solution, viz., $(V'/V)^2 < 3(\omega+1)$ and $\omega_\phi \approx -1 + (V'/V)^2/3$ [13,16]. To achieve such a solution, we invoke the second term in our potential (1), which plays a dominant role in the dynamics for small values of ϕ/m_{pl} . The potential $V(\phi)$ has a minimum if $A\alpha/\beta < 0$. The dynamics in this case is quite different from the case when ϕ rolls down monotonically. We will analyze the two cases separately.

Let us first consider the case when $A\alpha/\beta > 0$, and the potential at present is dominated by the second term, i.e.,

$$V_{now} \approx \frac{\lambda \alpha^2}{4\pi A} \exp\left(\frac{\alpha \phi_e}{m_{pl}}\right) \exp\left(\frac{-\beta \phi_{now}}{m_{pl}}\right). \quad (23)$$

It can be seen from Eq. (21) that the tracking condition ($\Gamma = 1$) is satisfied. However, in the transient regime when both the terms in Eq. (1) are of comparable magnitude, one obtains $\Gamma = 1$ only for $\alpha/\beta \approx O(1)$. Since early universe dynamics constrains $-a \approx O(10^2)$, acceptable values for β in this case would violate the late time attractor requirement of $\omega_\phi \approx -1 + \beta^2/3$. So for this scheme to work, the present universe should be well out of the transient regime. In other words, the validity of Eq. (23) should be accurate.

The value of the potential at present V_{now} should be equal to the present energy density, $\rho_c \approx 10^{-47} (\text{GeV})^4$. Combining Eq. (17) or (18) with Eq. (22) one could set the average value of the quantity $[-\alpha(\phi_e - \phi_{now})/m_{pl}] \approx O(60)$. If we set the value of $\phi_{now}/m_{pl} \approx O(1)$, then equating V_{now} with ρ_c and using Eq. (11) one obtains

$$A \approx \frac{10^{80} \exp(\alpha - \beta)}{\alpha^4}. \quad (24)$$

That we require such a large value for A does not come as a surprise, since we have reached the present energy density starting from a scale $V_e \approx O(10^{15}/\alpha)$ GeV.

We now analyze the case if $A\alpha/\beta < 0$. Here one gets a minimum for the potential with

$$\frac{\phi_{min}}{m_{pl}} = \frac{\ln(-A\alpha/\beta)}{\alpha - \beta} \quad (25)$$

and

$$V_{min} = \frac{\lambda \alpha^2}{4\pi A} \exp\left(\frac{\alpha \phi_e}{m_{pl}}\right) (A(-A\alpha/\beta))^{-\alpha/(\alpha-\beta)} + (-A\alpha/\beta)^{-\beta/(\alpha-\beta)}. \quad (26)$$

For this case one does not require the second attractor solution, and hence no corresponding restriction on the value of β . Numerical integration in Ref. [16] confirms that when ϕ reaches the minimum of the potential, the effective cosmological constant V_{min} takes over and oscillations are damped, thus driving the equation of state towards $\omega_\phi = -1$. For our model, the requirement that $V_{min} \approx V_{now}$ can be satisfied for $\phi_{min}/m_{pl} \approx O(1) \approx A$, and $-\alpha \approx 10^2 \approx \beta$. However, again setting $[-\alpha(\phi_e - \phi_{now})/m_{pl}] \approx O(60)$, the tuning required in this case is given by

$$\frac{-\alpha}{\beta} \approx 1 - O(10^{-60}) \alpha^3. \quad (27)$$

We thus find that the basic requirements for viable scenarios of quintessence are possible for certain choices of the parameters. It remains to be seen if observations indicate whether or not a potential with a minimum is favored. With the availability of more precise data in the future, the accurate reconstruction of quintessence potentials and the equation of state parameter may be possible. It is hoped that programs such as the ones initiated in Ref. [26] using red shift—luminosity distance correlations and in Ref. [15] using CMB data [27] will enable the enforcement of tighter constraints on the signature and value of the parameters A and β .

V. CONCLUSIONS

We have analyzed the dynamics arising from a scalar field rolling down the slope of an exponential potential in the framework of brane cosmology. The brane world inflationary scenario is feasible with steep potentials as distinct from the situation in standard cosmology. This is the case for both exponential potentials [8] and inverse power law potentials [7]. During inflation the desiderata of enough inflation and the COBE normalized amplitude of density perturbations are used to fix the values of the brane tension and the scale of the potential at this stage. The generic predictions of the models using exponential potentials is the parameter independence of the spectral index n and the tilt A_S^2/A_T^2 [8]. Subsequently,

the requirements of suppression of gravitino production and the emergence of radiation domination before nucleosynthesis constrain the effective slope of the potential after inflation. The tracker behavior of the scalar field in an exponential potential ensures the viability of dynamics in the matter dominated era.

The emergence of a second phase of accelerated expansion that we observe today necessitates the introduction of a second exponential term in the potential. From the point of view of construction of a model of just quintessence, there exists an ongoing debate as to whether [25] or not [16,28] a potential with a single exponential term is able to produce a viable scenario. However, our analysis shows that a combination of two exponential terms is essential for obtaining both inflation and present acceleration. The motivation for considering such a potential is largely phenomenological. Nevertheless, such kinds of potentials do arise in the conformal Einstein frame due to Brans-Dicke or other types of

nonminimal couplings of the scalar field. The inclusion of a higher dimensional quantum effect or a cosmological constant together with a four dimensional potential gives rise to effective potentials with two or more exponential terms [29].

In order to obtain viable inflation and radiation and matter dominated eras we ensure that one of the two terms in the potential dominates throughout these epochs. The values of the parameters must be so chosen that the second term starts playing a comparable role after the era of galaxy formation. In this way we are able to obtain a workable scheme of quintessence. To conclude, we obtain a scalar field dominated cosmology in the brane world scenario where one part of the potential drives inflation, and the other part plays a crucial role in quintessence. In order to achieve this with three parameters (α, β, A) in the potential, our results show that through the present status of analysis of observational data [4,11,27] one parameter (α) is constrained, another (either A or β) has to be tuned, and the sign of the remaining one is dictated by the choice of the other two.

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