Interacting black holes on the brane: the seeding of binaries

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Abstract

We consider the evolution of subhorizon-sized black holes which are formed during the high energy phase of the braneworld scenario. These black holes are long-lived due to modified evaporation and accretion of radiation during the radiation dominated era. We argue that an initial mass difference between any two neighbouring black holes is always amplified because of their exchange of energy with the surrounding radiation. We present a scheme of binary formation based on mass differences suggesting that such a scenario could lead to binaries with observable signatures.

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Cosmology in the braneworld scenario has inspired much activity in recent times. The feasibility of a large extra spatial dimension is the cornerstone of the Randall–Sundrum (RSII) braneworld model [1] in which all the standard model fields are confined to our observable 3-brane, except for gravity which can propagate also in the bulk. The braneworld description of our universe entails an early high energy phase during which the evolution of the universe is significantly altered. A particular feature of interest in RSII cosmology is the evolution of primordial 5-dimensional black holes. Black holes forming out of the collapse of horizon-sized density perturbations during the high energy phase obey a different evaporation law [2] compared to 4-dimensional black holes. It has been shown recently that accretion from the surrounding radiation bath can dominate over evaporation during the high energy phase [3]. This leads to prolonged survival of these primordial black holes with multifarious cosmological and astrophysical consequences. There exists the possibility that these black holes could be a significant fraction of cold dark matter. Several observational constraints on primordial black holes get modified in the braneworld scenario, as has been shown recently [4].

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Black holes as dark compact objects in galaxy haloes are the target of many recent and ongoing observations [5]. A large number of these black holes are expected to be in the form of binaries. Several interesting observational evidences ranging from lensing effects to hypervelocity star ejections have been put forward to bolster claims in support of black hole binaries [6] in a wide mass spectrum. Recently, it has been proposed that sublunar mass binaries comprised of braneworld black holes could produce detectable gravitational waves in their coalescing stage [7]. Gravitational lensing experiments do indeed leave open the possibility of the existence of sublunar mass black holes in certain mass ranges [8]. A formation mechanism for binaries in standard cosmology based on the inhomogeneous spatial distribution of primordial black holes has been proposed [9].

In this Letter we present a scheme of binary formation for primordial braneworld black holes based on mass differences. We consider subhorizon sized black holes in the RSII model formed during the high energy phase whose evolution has been studied in Refs. [2,3]. We shall consider the interaction of two neighbouring black holes which are initially well separated for their gravitational attraction to be approximated by Newtonian dynamics. The physical distance between two such neighbours increases with the Hubble expansion of the high energy radiation dominated era. These black holes exchange energy via the processes of evaporation and accretion with the radiation bath, and through it, with each other. The initial mass ratio of two such black holes is on average proportional to the square of the ratio of their formation times [2]. We will see from the evolution that the initial mass difference between two such black holes always increases during the radiation dominated era. We will then argue that such mass differences will facilitate the formation of binaries during the standard low energy phase via three-body gravitational interactions. We will consider an example of sublunar mass binary [7] formation through this scheme.

Let us first briefly state some essential results of Refs. [2,3]. The mass of a primordial braneworld black hole formed at time t_0 with initial mass m_0 during the high energy phase ($t < t_c$ with $t_c \equiv l/2$) grows due to accretion (evaporation is negligible, except for black holes with very small initial mass, or very low accretion efficiency f, i.e., except for $(M_0/M_4)^2 < 2A/(2B-1)(t_0/t_c)$, where $A \simeq 3/((16)^3\pi)$, and $B = 2f/\pi$, with $0 \leqslant f \leqslant 1$) such that

$$\frac{m(t)}{m_0} \simeq \left(\frac{t}{t_0}\right)^B. \quad (1)$$

This equation holds up to $t \simeq t_c$ assuming radiation domination persists, i.e., the radiation density $\rho_R = (3m_4^2)/[32\pi(t_c+t)t]$ stays greater than the sum of the energy density in all black holes, ρ_{BH}. The formation time is related to the initial mass by

$$\frac{t_0}{t_4} \simeq \frac{1}{4}\left(\frac{m_0}{m_4}\right)^{1/2}\left(\frac{l}{l_4}\right)^{1/2} \quad (2)$$

with the 4-dimensional Planck mass (time) labelled as m_4 (l_4). Assuming accretion to be effective only during the high energy phase, these 5-dimensional black holes evaporate as

$$m(t) = \left(m_{\max}^2 - 2Am_4^2\frac{t}{t_c}\right)^{1/2} \quad (3)$$

during the standard low energy regime $t \gg t_c$, with $m_{\max}$ being the maximum mass a black hole reaches via accretion. (Recently, there has been a conjecture of rapid evaporation through conformal modes for large braneworld black holes with size greater then the AdS radius l [10]. However, our focus will be on much smaller black holes.)

Now let us consider the evolution of two neighbouring black holes, BH1 and BH2, having masses m_1 and m_2 respectively, with initial masses $m_{1_0} < m_{2_0}$, or $t_{1_0} < t_{2_0}$. BH2 forms at a physical distance d_0 from BH1. Their physical separation d grows at a rate $\propto t^{1/4}$. We shall assume that $d_0 > l$, and that the gravitational potential $\phi = [m/(m_4^2 d)][1 + (2l^2)/(3d^2)] \ll 1$ so that exchange of energy between them via gravitational waves can be neglected during the high energy regime. The change of mass of each black hole is effected by the net sum of its Hawking evaporation depending on its temperature, and the accretion of radiation depending on the mean radiation density in the universe [3]. Moreover, due to the presence of BH2, the net rate of change of the mass $\dot{m}_1$ of BH1 gets a further contribution proportional to the product of $\dot{m}_2$ and the solid angle subtended by BH2 on BH1, and vice versa. In other words, BH1 feels the local difference from the mean radiation density as a result of an accreting (or evaporating) BH2, and similarly for

BH2. Thus the evolution equation for each of them can be written as

$$\dot{m}_i = \frac{Bm_i}{t} - \frac{Am_4^2}{m_i t_c} - \frac{r_i^2 \dot{m}_j}{4d^2} \tag{4}$$

for $i, j = 1, 2$ ($(r_i/l_4) = (8/3\pi)^{1/2}(l/l^4)^{1/2}(m_i/m_4)^{1/2}$ is the Schwarzschild radius for 5-dimensional black holes [2]). The first and second terms on the r.h.s. represent accretion from the average radiation density and evaporation, respectively [3]. The last term arises from the local inhomogeneity in radiation density due to the jth black hole. Considering both BH1 and BH2 to be in their accreting phases at the formation time t_{0_2} of BH2, it is possible to see from Eqs. (4) that for BH1 the first term is comparable to the interaction (third) term when $(d/r_2) \sim ([t-t_{1_0}]/[t-t_{2_0}])^{1/2}$. Thus, for a while after its formation, BH2 is able to suppress the growth of BH1. If on the other hand, BH2 forms at a time when BH1 begins to evaporate, with or without the aid of the interaction, BH2 registers enhanced growth due to the locally denser radiation coming from the evaporating BH1.

The effect of two interacting black holes on their evolution can be more precisely formulated as follows. Defining $\tilde{t} = Am_4^2 t/t_c$, Eqs. (4) can be written as

$$\dot{m}_i = \frac{Bm_i}{\tilde{t}} - \frac{1}{m_i} - g\frac{m_i \dot{m}_j}{\tilde{t}^{1/2}}, \tag{5}$$

where

$$g = \frac{4A^{1/2}}{3\pi}\left(\frac{l_4}{d_0}\right)^2\left(\frac{t_0}{t_4}\right)^{1/2}\left(\frac{t_c}{t_4}\right)^{1/2} \tag{6}$$

can be regarded as a coupling constant between BH1 and BH2 dependent on their initial separation d_0. Now, in terms of the variables $s = \ln(\tilde{t}/t_0)$ and $x_i = m_i/\tilde{t}^{1/2}$, Eq. (5) can be cast in an autonomous form, i.e.,

$$x_i' = \left[B - \frac{1}{2} - g\left(\frac{x_j}{2} - x_j'\right)\right]x_i - \frac{1}{x_i}, \tag{7}$$

where accents denote derivatives with respect to s. In the following we confine our attention to small enough values of g, such that $g^2 x_1 x_2 < 1$. This is indeed the condition for Eq. (7) to be well behaved. For two equal mass black holes ($x_1 = x_2 = x$), this condition becomes $g < g_\star = 1/x$. This limit coupling strength in fact corresponds to the limit of coalescence, since the black hole radius r is of the order of the separation d for $g = g_\star$. Below the critical value $g_c = [2(2B-1)^3/27]^{1/2}$ of the coupling constant, the dynamics of two equal mass black holes has a stable fixed point x_F. Our interest is in two unequal mass black holes which are initially well separated for gravitational interaction to be subdominant. If one considers a small initial mass difference, then the two black hole masses will evolve as $x_1 = x(s) - \epsilon(s)$ and $x_2 = x(s) + \epsilon(s)$, with $\epsilon(s) \approx \epsilon(0)\exp(\int_0^s \Lambda(x(u))\,du)$, where the instantaneous Lyapunov exponent

$$\Lambda(x) = \frac{2B-1}{2} + \frac{1 + 2gx + Bg^2x^4}{x^2(1-g^2x^2)} \tag{8}$$

is larger than the constant $(2B-1)/2$. It follows that any small initial mass difference blows up exponentially in the first stages of dynamical evolution. In particular, the symmetric fixed point x_F is always linearly unstable against a perturbation of the form $\delta x_1 = -\delta x_2$.

It is thus clear that the physical solutions of Eqs. (5) exhibit disequilibration for black holes masses, i.e., if for two neighbouring black holes, $m_1(t_0) < m_2(t_0)$, then $m_1(t) \ll m_2(t)$ for $t > t_0$. Requiring the era t with a number density n_{BH} of black holes of average mass $m(t)$ to be radiation dominated, i.e., $\rho_{\mathrm{BH}} \equiv mn_{\mathrm{BH}} \leqslant \rho_R$, the mean strength of the coupling $\bar{g}$ between two such black holes should be such that

$$\bar{g} \leqslant \frac{A^{1/2}}{4}\left(\frac{m_4}{m_0}\right)^{2/3}\left(\frac{t_4^2}{t_0 t_c}\right)^{1/6} \ll g_\star. \tag{9}$$

In general however, there could be spatial inhomogeneities in the formation of black holes. The growth rate of any given black hole is hence impacted by the mass spectrum and spatial distribution of all neighbouring black holes. For a system of two black holes BH1 and BH2 formed at eras t_{1_0} and t_{2_0} with the initial mass difference $(\Delta_{12})_0 = m_{2_0} - m_{1_0}$, and when both are in their accreting phases, the mass difference at time $t \gg t_{2_0}$ is given by (from Eqs. (1) and (2))

$$\begin{aligned}(\Delta_{12})(t) - (\Delta_{12})_0 \\ \geqslant m_{2_0}\left[\left(\frac{t}{t_{2_0}}\right)^B\left(1 - \left(\frac{m_{1_0}}{m_{2_0}}\right)^{1-B/2}\right) \right. \\ \left. - \left(1 - \frac{m_{1_0}}{m_{2_0}}\right)\right]\end{aligned} \tag{10}$$

with the equality sign holding for $g = 0$, i.e., when the interaction term can be neglected.

For the present analysis, we are assuming that black hole accretion is restricted to the high energy phase. Hence, when the universe exits the high energy phase, the masses of BH1 and BH2 are frozen to m_1 and m_2 (say). Hawking evaporation continues to be negligible for a long time beyond t_c[3]. Consider a spherical region of radius d in which the average matter density (contributed by BH1 and BH2) is given by

$$\bar{\rho}_{\rm BH} = \frac{3(m_1+m_2)}{4\pi d^3}. \tag{11}$$

Since radiation density in this region falls off as a^{-4}, the sphere of radius d will become matter dominated at a time t_f given from Eqs. (1) and (2) by

$$t_f \simeq 2\left(\frac{8}{3\pi}\right)^3 \left(\frac{d_0}{r_{2_0}}\right)^6 \left(\frac{t_0}{t_c}\right)^{2B+1/2} t_c. \tag{12}$$

For $t > t_f$ the matter dominated region with radius d gets cutoff from the background radiation dominated expansion which continues up to the era of matter-radiation equality $t_{\rm eq} \gg t_c$. For $t > t_f$, the black holes BH1 and BH2 will form a bound system [9]. Assuming that a bound system (or binary) gets formed only in the low energy regime ($t_f > t_c$), it follows from Eq. (12) that

$$\frac{d_0}{r_{2_0}} > \frac{1}{64}\left(\frac{3\pi}{8}\right)^{1/2} \left(\frac{t_c}{t_{2_0}}\right)^{\frac{4B+1}{12}}. \tag{13}$$

With the initial density fraction $\alpha_{m_0} (= \rho_{\rm BH}(t_0)/\rho_R(t_0))$ for black holes formed with mass m_0, the mean initial separation is $\bar{d}_0 = (3\pi/8)^{1/2}(r_0/\alpha_{m_0}^{1/3})$. (Two black holes with initial separation less than that given in Eq. (13) will form a bound system in the high energy phase. Accretion of radiation will be negligible after binary formation, with a resultant smaller lifetime. In the present analysis we shall not consider details of such a scenario.) The size of this bound system d_f is related to the initial black hole separation by $d_f = d_0(t_c/t_0)^{1/4}(t_f/t_c)^{1/2}$. Using Eqs. (1), (2) and (12),

$$d_f = \frac{1}{16}\left(\frac{t_c}{t_0}\right)^{3-B} \left(\frac{d_0}{l}\right)^3 d_0 \tag{14}$$

with its mean value given by $\bar{d}_f = (4/\alpha_{m_0}^{4/3}) \times (t_0/t_c)^{1+B} l$.

Binary formation proceeds as a result of 3-body interaction. Let us choose three neighbouring black holes to have mass differences at t_c given by $m_2 - m_1 = \Delta_{12}$ and $m_3 - m_2 = \Delta_{23}$. Such a scenario will arise generically since the initial mass differences are due to horizon sized collapse at different eras, and possible interactions through the radiation background can only amplify such mass differences between neighbouring black holes during the high energy regime. BH2 will become gravitationally bound to BH3 at time t_f with an elliptical orbit whose major axis

$$a_f \propto d_f. \tag{15}$$

The role of BH1 is to provide the tidal force required to prevent head-on collision between BH2 and BH3. The minor axis b_f of the binary is proportional to the tidal force times the squared free fall time [9], i.e.,

$$b_f \propto 2\left(\frac{m_2-\Delta_{12}}{2m_2+\Delta_{23}}\right)\left(\frac{d_f}{d_P}\right)^3 a_f, \tag{16}$$

where d_P is the perpendicular distance of BH1 from the mid-point of the BH2–BH3 axis. Although BH2 binds to BH3 at time t_f, mass differences hamper the possibility of BH1 being gravitationally bound to BH3. Even if the three black hole separations are equal before binary formation, the total region encompassing the binary and BH1 will become matter dominated at a time $(t_{\rm md})_{13}$ much later than t_f, given by $(t_{\rm md})_{13} \geqslant t_f[(m_{3_0}+m_{2_0})/(m_{1_0}+m_{2_0})]^{11/4-B}$, (since using Eqs. (2) and (11), $t_f \propto m_0^{B-11/4}$) with the equality sign signifying $g = 0$. But during the interval $t_f < t < (t_{\rm md})_{13}$, BH1 continues to move away from the BH2–BH3 binary with the Hubble flow, i.e., d_P registers growth by a factor $[(t_{\rm md})_{13}/t_f]^{1/2}$. The gravitational interaction between the binary and BH1 at $t = (t_{\rm md})_{13}$ is much weakened compared to the strength of the BH2–BH3 interaction.

The eccentricity of the binary orbit can be obtained from Eqs. (15) and (16). For uniformly distributed black holes formed around the era t_0 with masses $\sim m_0$, the mean eccentricity is given by

$$\bar{e} = \left[1 - \eta^2 2\left(\frac{m_2-\Delta_{12}}{2m_2+\Delta_{23}}\right)^2\right]^{1/2}, \tag{17}$$

where η is an $O(1)$ geometrical factor [9]. The Newtonian approximation that we have used for describing the gravitational interaction between black holes forming a binary is valid if the gravitational potential $|\phi| \ll 1$, which for the BH2–BH3 binary reads

$(m_1/m_4^2) \ll d_f(1-e)$. Using Eqs. (1), (14) and (17) one can verify that the Newtonian analysis holds good for $(t_4/t_0)^{2B-1} \ll \eta^2 \alpha_{m_0}^{-4/3}[(m_2-\Delta_{12})/2m_2+\Delta_{23})]^2$.

As a particular example of the scenario outlined by us, consider three black holes formed about the era $t_0 \sim 10^{26} t_4$ with average mass $m_0 \sim 10^{24} m_4$, radius $r_0 \sim 10^{27} l_4$ and $\Delta \sim O(m_1)$. We have chosen $l = 10^{30} l_4$, about the maximum allowed by present experimental bounds. Setting the accretion factor $B = 1/2$, one gets $m_{\max} \simeq m(t_c) = 10^{26} m_4 \sim 10^{21}$ g. The closure bound for a population of black holes with $m_0 \sim 10^{24} m_4$ can be obtained using the prescription of Clancy et al. [4], and turns out to be $\alpha_{m_0} < 6 \times 10^{-18}$ for the initial energy fraction. Choosing the maximum allowed value of α_{m_0} the mean initial separation is $\bar{d}_0 \sim 10^6 r_0 \sim 1$ cm. Nevertheless, the validity of the Newtonian approximation $|\phi| \ll 1$ is readily ensured for a choice $d_0 \sim 10^3 r_0$ for the specific system of three black holes we are considering for binary formation. For these parameters, binary formation takes place at $t_f \sim 10^{12} t_c$, with the major axis $a_f \sim 10^7$ cm. Note here that we have estimated the mean values for the above parameters assuming a uniform distribution of black holes. Inhomogeneities in the spatial distribution for black holes would change these values on a case-to-case basis. However, at least some of the binaries would have parameters close to the mean values. These would then fall in the category of sublunar compact objects discussed in Ref. [7], and gravitational waves emitted during their coalescence are within the observational range of third generation gravitational wave detectors.

To summarize, we have studied the effect of interaction of horizon-sized black holes formed during the radiation dominated high energy phase of the RSII braneworld scenario. We have seen that the processes of black hole evaporation and accretion mediated via the radiation bath causes the mass difference between two neighbouring black holes to be always amplified during the high energy era. We have presented a scheme by which binary formation takes place through three-body gravitational interaction during the standard low energy era. This scheme of binary formation based on mass differences could be viewed as an additional possible mechanism to the scheme based on spatial inhomogeneities [9]. In a realistic scenario, both spatial and mass inhomogeneities are expected in any given configuration of black holes, and these separate effects will interfere to either abet or inhibit binary formation depending upon the particular configuration. The specific example we have furnished suggests that the parameters for such binaries might lie within the range of sublunar compact objects amenable for detection by forthcoming observations of gravitational waves such as by the EURO detector [7]. It is important to note that we have considered black hole accretion only during the high energy phase. If accretion of radiation continues during the standard low energy regime up to t_{eq}, as claimed in Ref. [11], it might be worthwhile to investigate the possible formation of more massive binaries through primordial black holes. One should keep in mind that black holes also grow by accreting mass from the surrounding matter distribution in the matter dominated era. Finally, the possibility of accreting the homogeneous and presently dominating cosmological or scalar field energy [12], if realized, could push up the masses further.

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